Grey-box Modelling of a Quadrotor Using Closed-loop Data

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Saab Aeronautics

Aerospace Technology Congress 2016
2016-10-12

Introduction
The Quadrotor
System Identification
Grey-box Modelling
Closed-loop Identification
Two-stage method
Result
Conclusions

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2. System Identification
3. Result
4. Conclusions

Introduction

Background

In control design, there is a need of a model describing the dynamics of the system.
Often, a physical model is complex and/or inaccurate due to simplifications.

Goal

Find a simple mathematical model describing the dynamics of a hovering quadrotor.

Approach

Use measured input and output data to estimate a model: System Identification

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The Quadrotor

System Overview

Photo: M.Blomberg

- Accelerometer
- Gyroscope
- Magnetometer

- Voltage to motors
- Joystick reference signals

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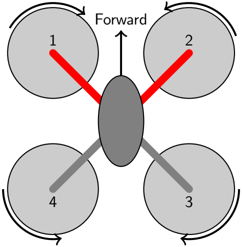
The Quadrotor

Maneuverability

- Four control signals, one for each motor
- Maneuverability due to thrust differences
- Roll input:
  $u_{\phi} = \frac{1}{4}\left(u_1 + u_4 - u_2 - u_3\right)$



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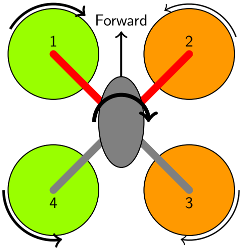

System Identification

Main idea

An approach to create a mathematical model of a system

- Assume one or more mathematical structures (equations)
- The structures have a set of unknown values, called parameters (Θ)
- Estimate Θ using collected input-output data

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System Identification

Grey-box Modelling

Black-box Modelling

- Estimated from input-output data
- No connection to the underlying physics
- Model restricted by order

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- Structure from physical relations
- Physical meaning
- Model restricted by structure

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$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \Theta_1 & \Theta_2 \\ \Theta_3 & \Theta_4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \Theta_5 \\ \Theta_6 \end{pmatrix} u$$

$$y = \begin{pmatrix} \Theta_7 & \Theta_8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

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$$\begin{pmatrix} \dot{p} \\ \dot{T}_\phi \end{pmatrix} = \begin{pmatrix} \Theta_1 & \Theta_2 \\ \Theta_3 & \Theta_4 \end{pmatrix} \begin{pmatrix} p \\ T_\phi \end{pmatrix} + \begin{pmatrix} \Theta_5 \\ \Theta_6 \end{pmatrix} u_\phi$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} p \\ T_\phi \end{pmatrix}$$

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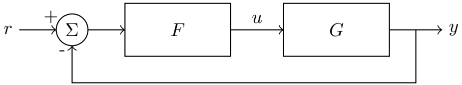
Parameters estimated using the Prediction-Error Method (PEM)

Closed-loop system

A system with output feedback.



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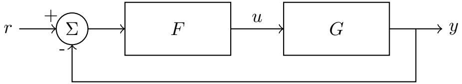
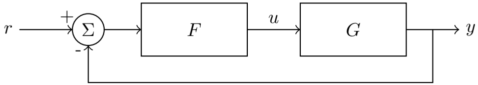


Approaches to closed-loop identification

- Direct approach
  - First approach
  - Ignore the feedback
- Take the controller into consideration
  - For example: the *Two-stage method*

System Identification

Two-stage method



1. Estimate a model $\hat{S}$ from $r(t)$ to $u(t)$

$$\hat{S} \approx \frac{F}{1+FG}$$

2a. Simulate the input: $\hat{u}(t) = \hat{S}r(t)$

2b. Use the **simulated** input and **measured** output to estimate the model $\mathcal{M}$

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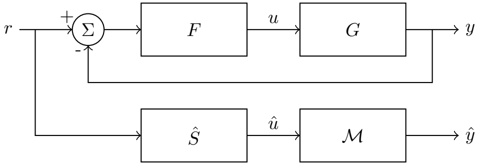

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Roll Rate Model

Resulting Model

- Models estimated using both the direct approach and the two-stage method
- Similar results from both methods
- Direct approach model chosen due to simplicity

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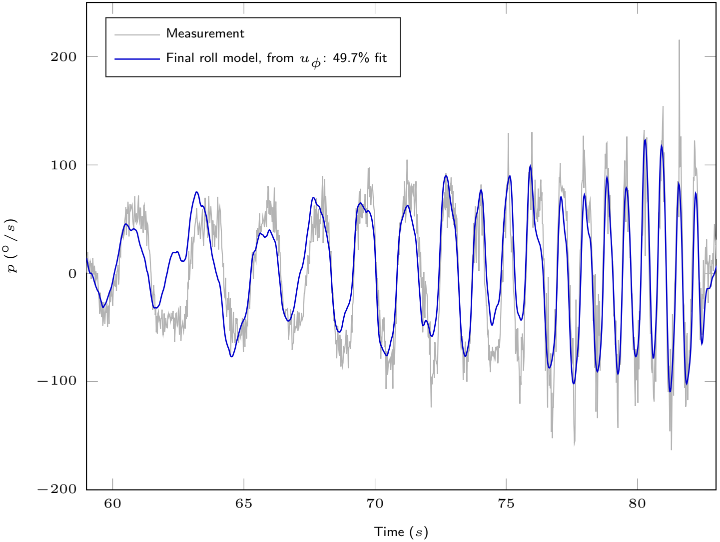
Roll Rate Model

Simulation from u_ϕ



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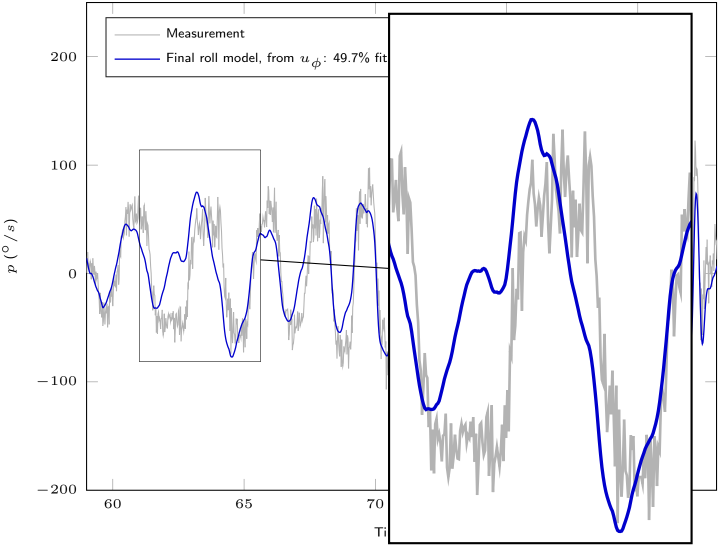

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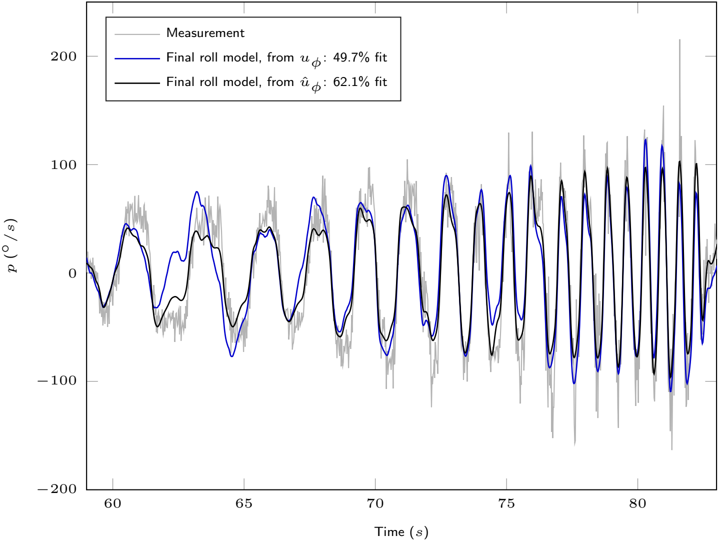

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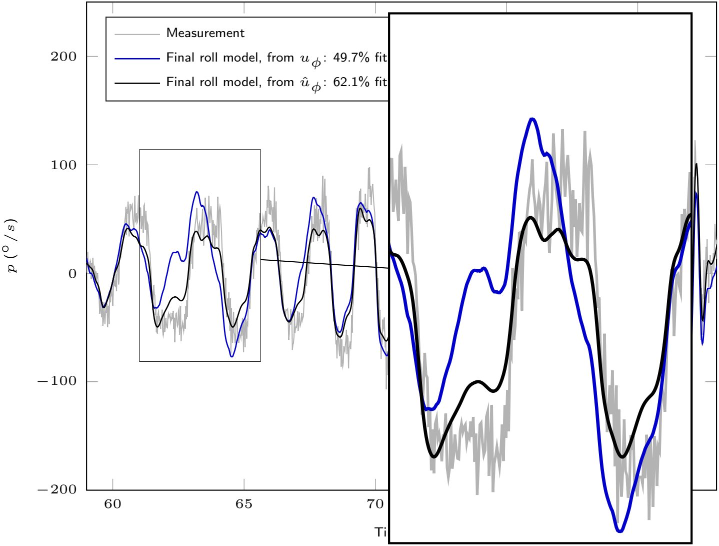

Conclusions

- The direct approach is good enough.
- The two-stage method provides insights to the closed-loop system.
- For control design, this model is good enough.

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The End

Thank you!