# Derivation of world's largest $K_I$-data base for twin cracks at countersunk holes

by

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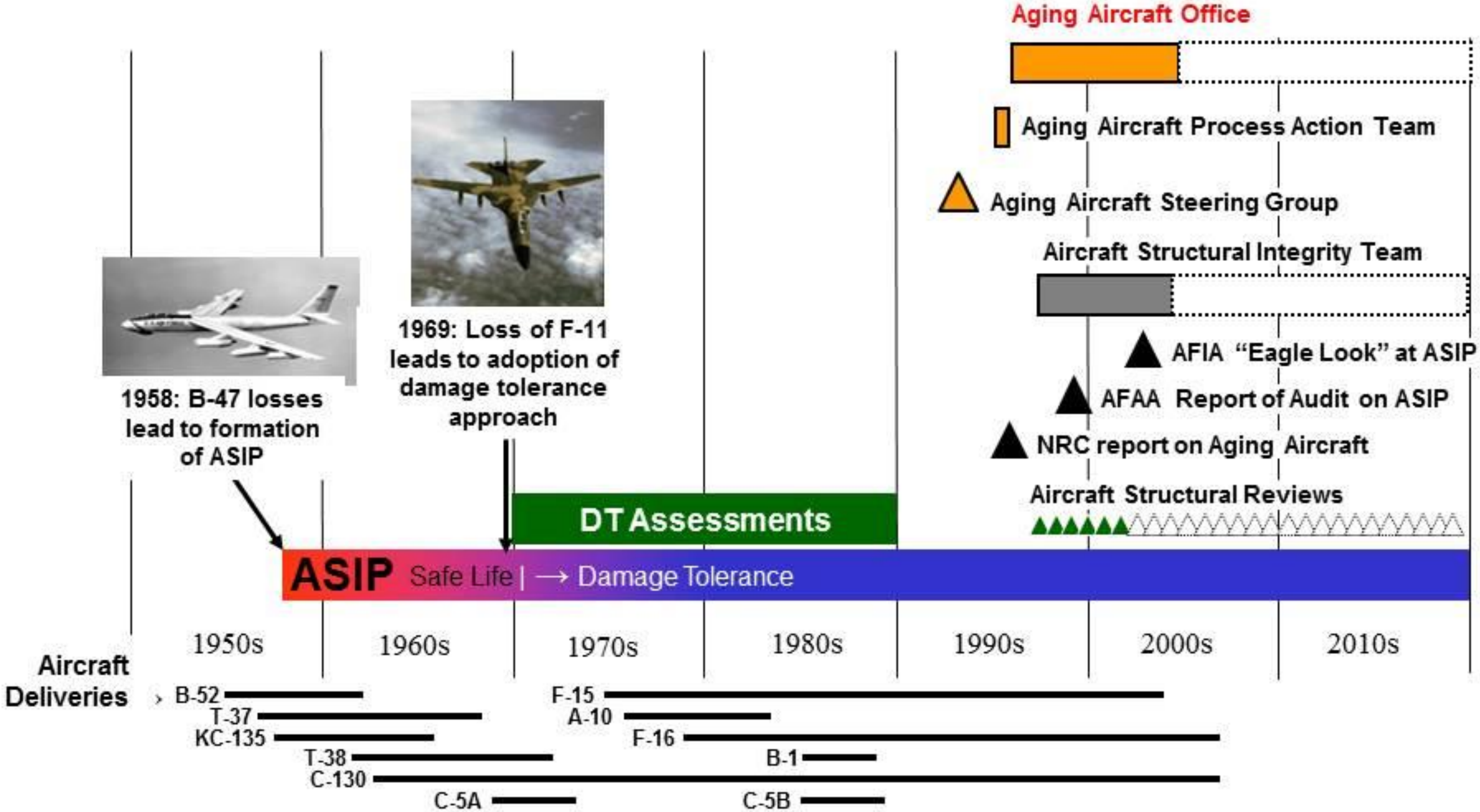
# Outline

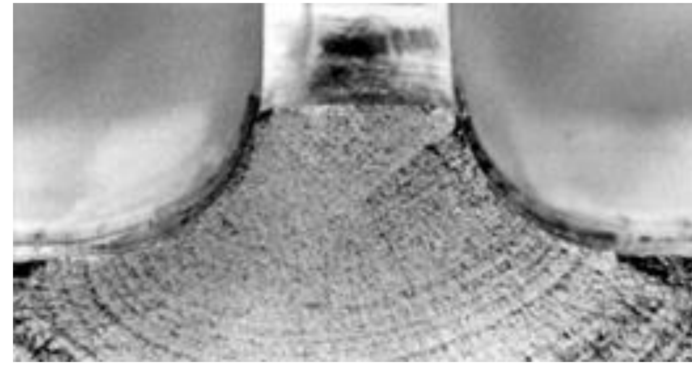
- Introduction
- ASIP Aircraft Structural Integrity Program
- Fatigue Crack Growth Models, Software and $K_I$ - Data Bases
- Existing $K_I$ - Data for Countersunk Hole Geometries
- Creation of a Data Base with (61*M*) $K_I(a/t, c/a, b/t, R/t)$ functions
- 3D *hp*-version of FEM
- Accurate Calculation of $K_I(\phi)$ at arbitrary $\phi = \phi^*, 0 < \phi^* < \bar{\phi}(a,c,b)$
- Meshes (*hp*-FEM) for analysis of Cracks at Countersunk Holes
- Fast convergence of $K_I(\phi), \phi = \phi^*, 0 < \phi^* < \bar{\phi}$ to relative errors $< 10^{-4}$
- A mathematical splitting method for fast $K_I$ -analysis of multiple crack
- A HPC-based System for generation of large $K_I$ - Data Bases
- Results (i.e. 20 graphs of 61*M*)
- Summary

# History of Structural Integrity Efforts

*Introduction*

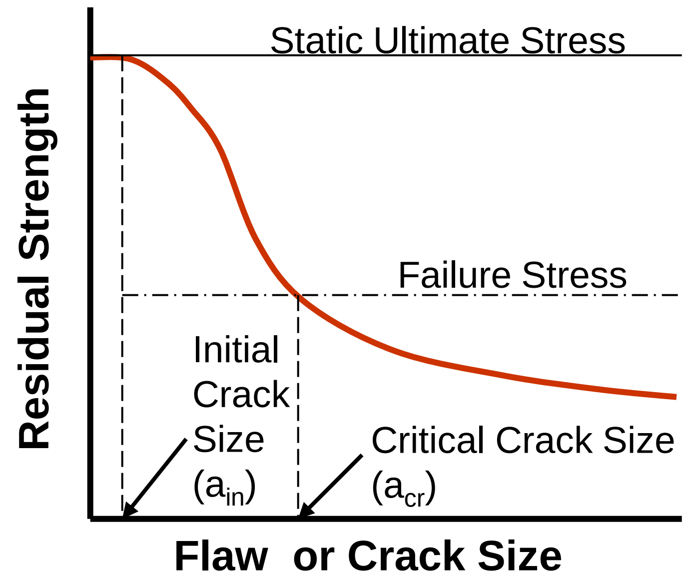


# ASIP in Practice



- ***Fatigue failure*** – failure caused by ***cyclic*** loading
- Load cycles often cause the growth of cracks from
  inherent metallurgical features or from damage induced during manufacturing or service

- As a fatigue crack grows in a component, the component loses the ability to withstand stresses
- Thus, fatigue crack growth also causes a loss of ***Residual Strength***




**Analytical fatigue crack growth/failure prediction**

- Paris Model $$\frac{da}{dN} = C \cdot \Delta K_I^m(a)$$

- Failure Criterion $$|K_c| < |K_{I,Applied}|$$

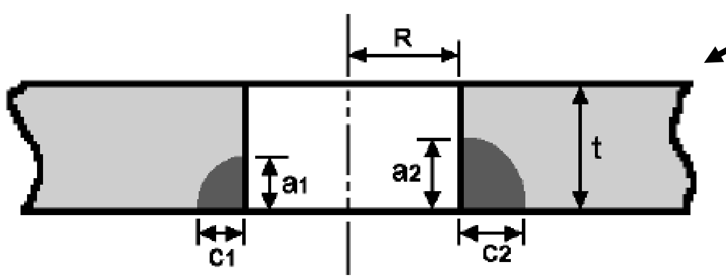
$C$, $m$, $K_c$ are material dependent parameters

# Fatigue Crack Growth Models, Software and $K_I$ - Data Bases

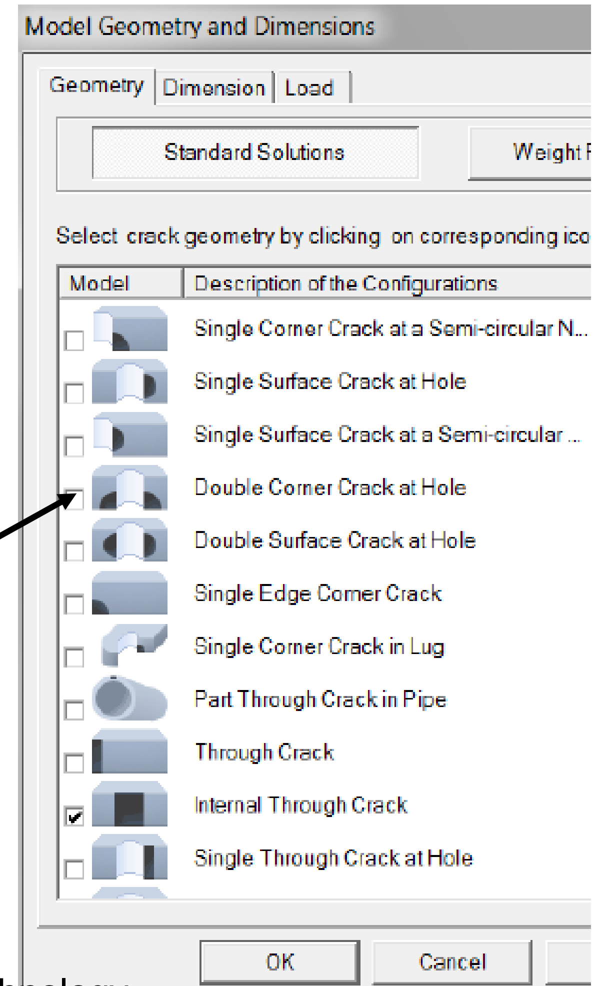
Forman and Newman at NASA, De Koning at NLR, and Henriksen at ESA developed the elements of the NASGRO (Version 3.0) crack growth rate equation. It has been implemented in AFGROW as follows:

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{max}}{K_{crit}}\right)^q}$$

Where *C, n, p* and empirically derived.



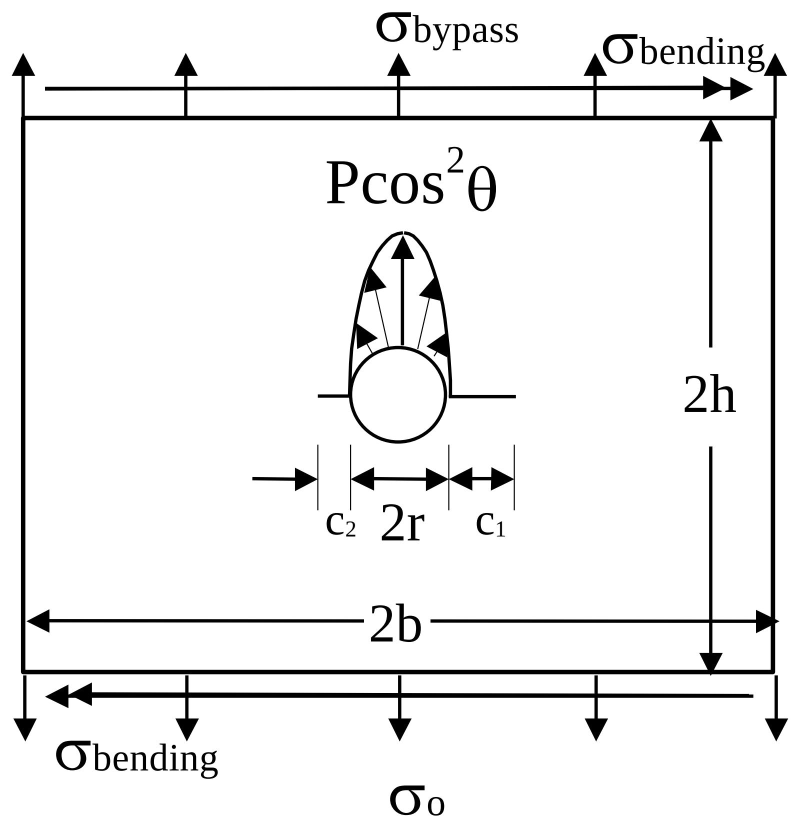

Corner cracks at straight shank hole

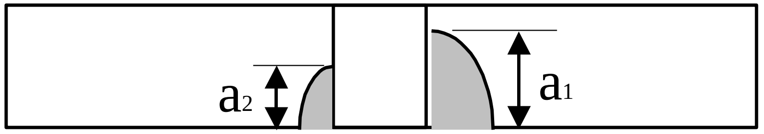


# $K_I$ - Data Bases 1

***5.7M* solutions delivered to users via *AFGROW (2003)* and *NASGRO (2012)***
***(S. Fawaz and B. Andersson)***

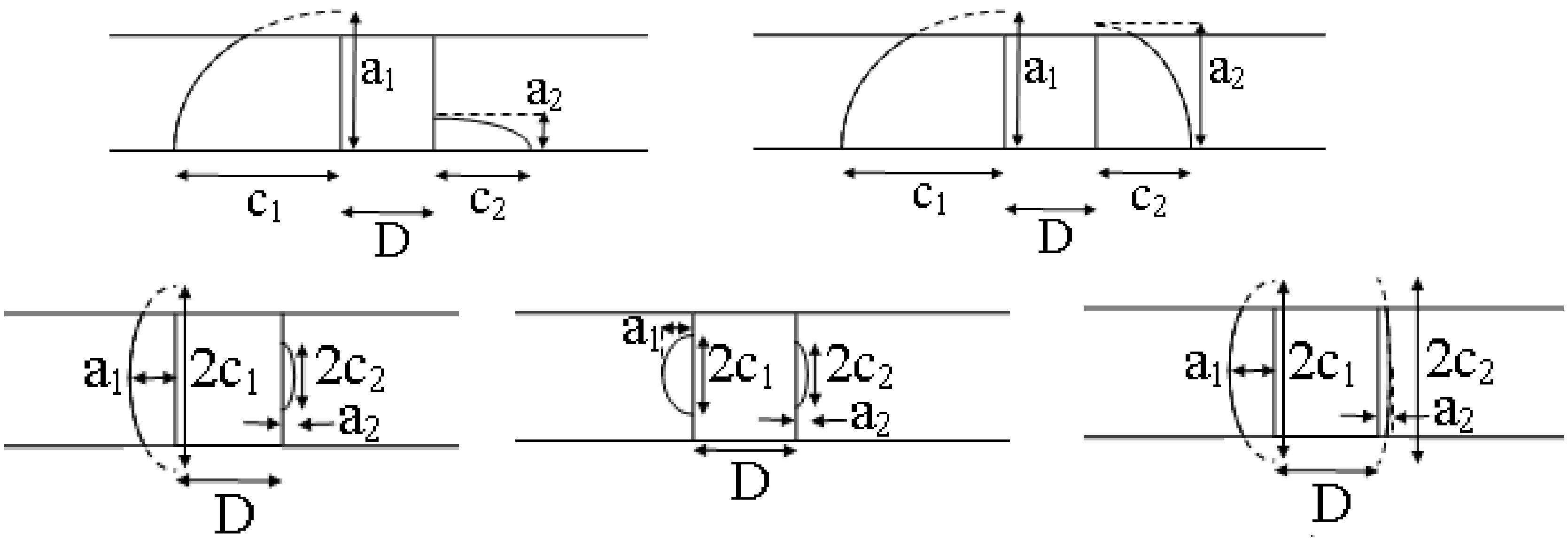
- Geometry
  - Centrally Located Straight Shank Hole
  - $0.1 \leq r/t \leq 10.0$
    - 0.1, 0.111, 0.125, 0.1428, 0.1667, 0.2, 0.25, 0.333, 0.5, 0.667, 0.75, 0.8, 1.0, 1.25, 1.333, 1.5, 1.667 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0
  - Finite Width/Height Plate
    - r/h = 0.0025
    - r/b = 0.0025
- Crack Shapes
  - $0.1 \leq a/c \leq 10.0$
    - 0.1, 0.111, 0.125, 0.1428, 0.1667, 0.2, 0.25, 0.333, 0.5, 0.667, 0.75, 0.8, 1.0, 1.25, 1.333, 1.5, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0
  - $0.1 \leq a/t \leq 0.99$
    - 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99
- Load Conditions
  - Tension
  - Bending*
  - Pin Loading (Bearing)



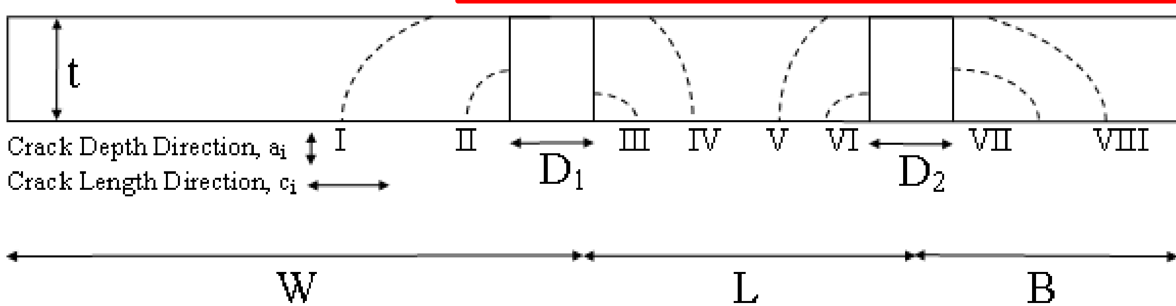
## $K_I$ - Data Bases 2

***2M*** **solutions delivered to** ***AFGROW (2014)***



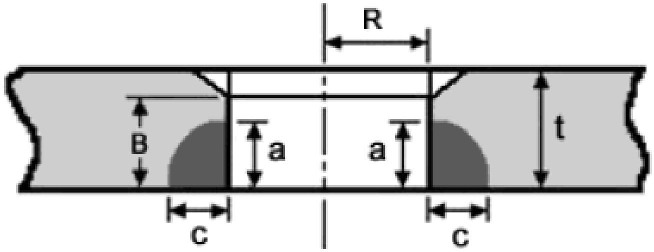

## $K_I$ - Data Bases 3

***2000M*** **solutions delivered to** ***AFGROW (2012)***



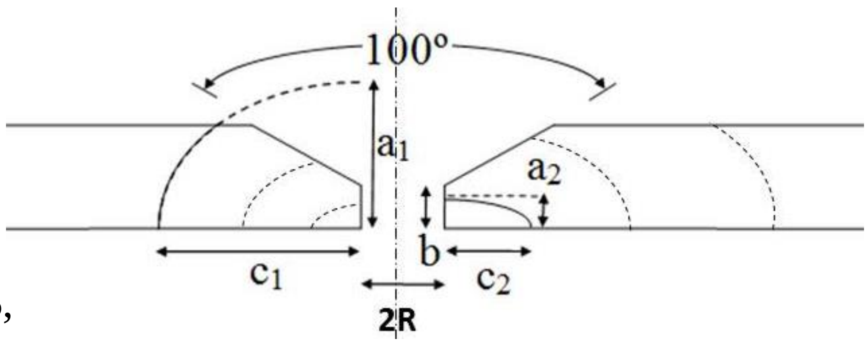
## $K_I$ - Data Bases 4

**3k crack scenario delivered to *AFGROW* by Dr Reinier de Rijck, NLR, 2014**




### Present Project: A Database with $K_I(c_1, c_2, a_1, a_2, b, R/t)$ -data

- **10.2*M* twin crack scenario**
- 34*k* single crack scenario
- Tension, bending and pin loading.



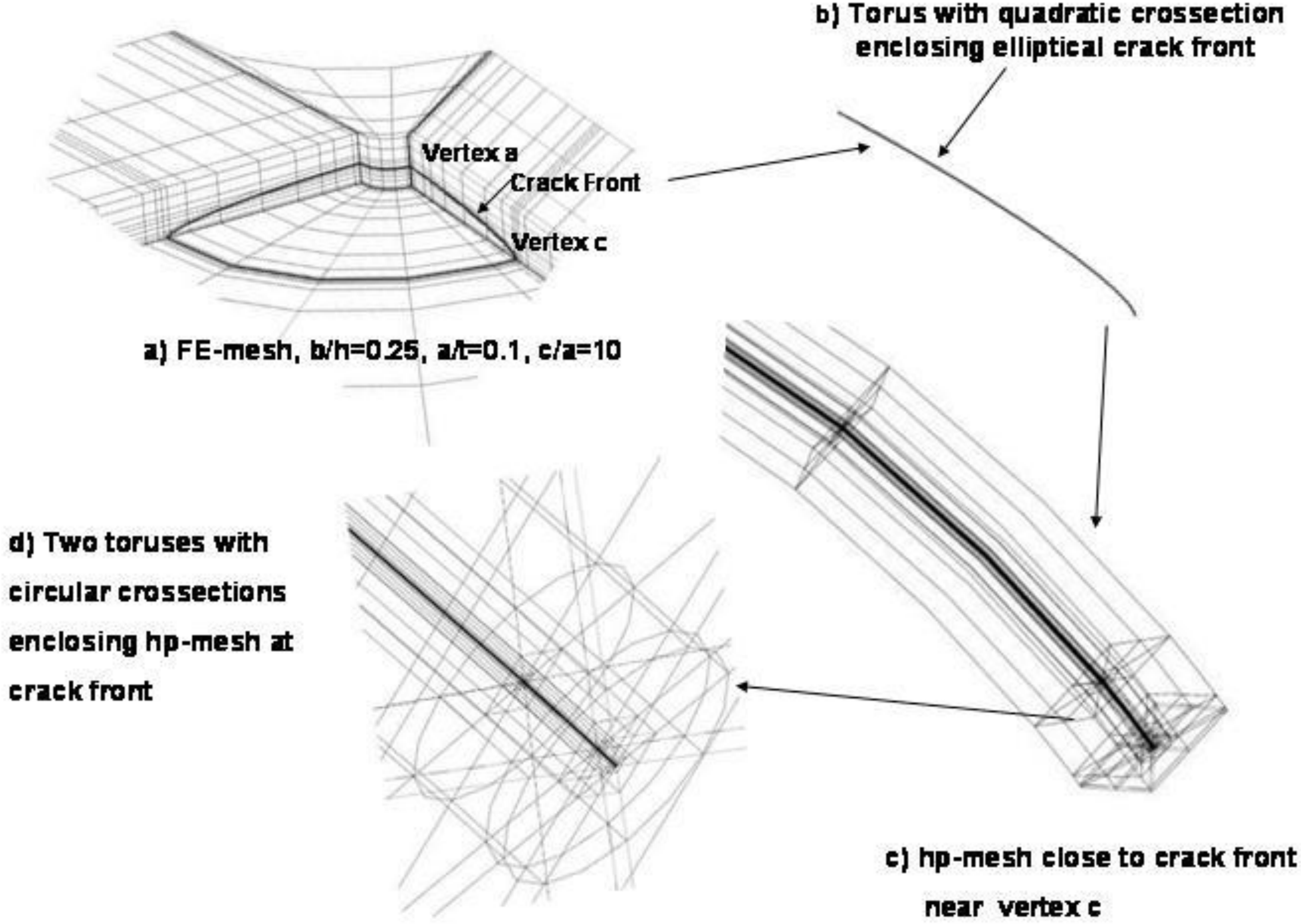

$c/a = 0.10,\ 0.1667,\ 0.333,\ 0.5000,\ 0.667,\ 0.75,$
$0.800,\ 0.900,\ 1.000,\ 1.111,\ 1.25,\ 1.333,\ 1.500,$
$2.00,\ 3.000, 4.00, 5.00, 6.00,\ 8.00,\ 10.00.$

$a/t = 0.10, 0.20, 0.30,\ 0.40, 0.50, 0.60, 0.70, 0.80,$
$0.90, 0.975, 1.05, 1.15, 1.25,\ 1.50, 1.75, 2.00, 2.50,$
$3.00, 4.00, 5.00,\ 6.00,\ 8.00, 10.00, \ldots, 800.00.$

$R/t = 0.10,\ 0.20,\ 0.50,\ 0.75,\ 1.0,\ 1.50,\ 2.0,$
$2.50,\ 3.0,\ 4.0,\ 5.0,\ 6.0,\ 8.0,\ 10.0.$

$b/t = 0.05,\ 0.25,\ 0.50,\ 0.75.$

# *hp*-version Meshes



# Accurate Calculation of $K_I(\phi)$ at arbitrary $\phi = \phi^*, 0 < \phi^* < \bar{\phi}$.

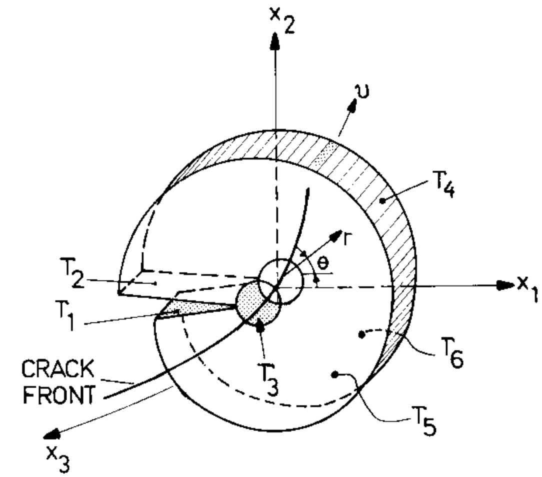
The displacements **u** at a point $x_3$ on the crack front can be written

$$u(r,\theta,x_3) = \sum_{\alpha=I,II,III} K_\alpha(x_3) r^{1/2} \Psi_\alpha(\theta) + \text{smoother terms}$$

For smooth edges, the edge intensity functions $K_\alpha(x_3)$ are analytic on open intervals $s_k \le x_3 \le s_{k+1}$. Hence, we approximate the edge intensity functions with the polynomials:

$$\bar{K}_\alpha(x_3) = \sum_{n=0}^{p} \bar{k}_{\alpha n} P_n(s), \qquad s = \frac{2(x_3 - s_k)}{s_{k+1} - s_k} - 1$$

Where $\bar{k}_{\alpha n}$ are unknown coefficients, $p$ is the polynomial order of the finite element trial functions, and $P_n$ the Legendre polynomials.



Domain $\Omega^e$ used for calculation of stress intensity factors

By applying the Maxwell-Betti reciprocity theorem the accuracy of the calculated *K*'s thus will depend only on a weighted average of the finite element solution inside the extraction domain. This gives *exponentially fast convergence*, with increasing *p* to the exact *K*-values.

## Point-wise convergence of $K_I(\phi_i)$

**P1**

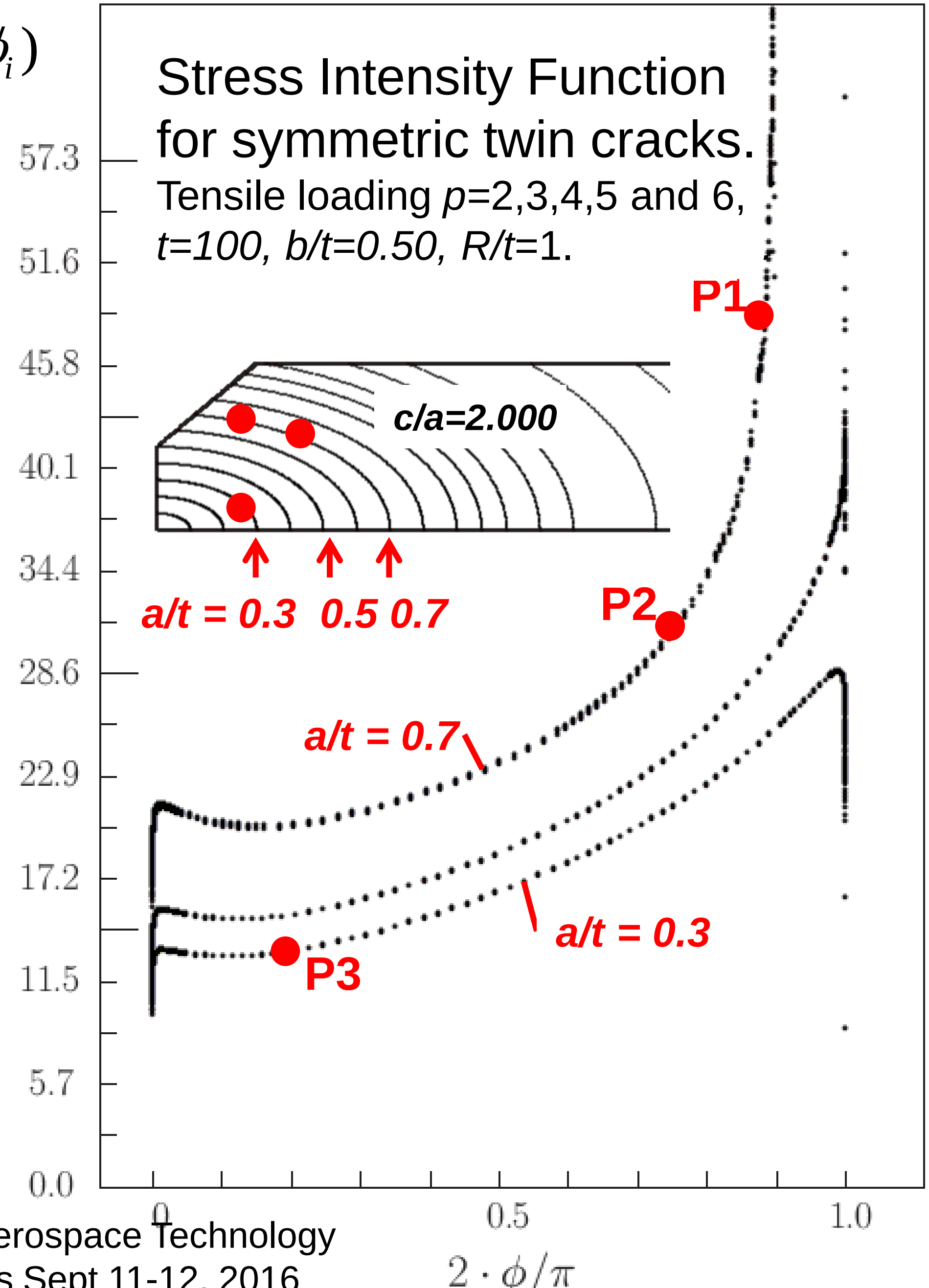
| p | $K_I$ | *Err %* |
|---|---|---|
| 2 | 48.18 | 0.75 |
| 3 | 47.91 | 1.31 |
| 4 | 48.50 | 0.10 |
| 5 | 48.55 | 0.02 |
| 6 | 48.55 | Zero |

**P2**

| p | $K_I$ | *Err %* |
|---|---|---|
| 2 | 31.25 | 0.44 |
| 3 | 31.07 | 1.01 |
| 4 | 31.37 | 0.08 |
| 5 | 31.39 | 0.02 |
| 6 | 31.39 | Zero |

**P3**

| p | $K_I$ | *Err %* |
|---|---|---|
| 2 | 12.87 | 0.40 |
| 3 | 12.84 | 0.14 |
| 4 | 12.81 | 0.07 |
| 5 | 12.82 | 0.02 |
| 6 | 12.82 | Zero |

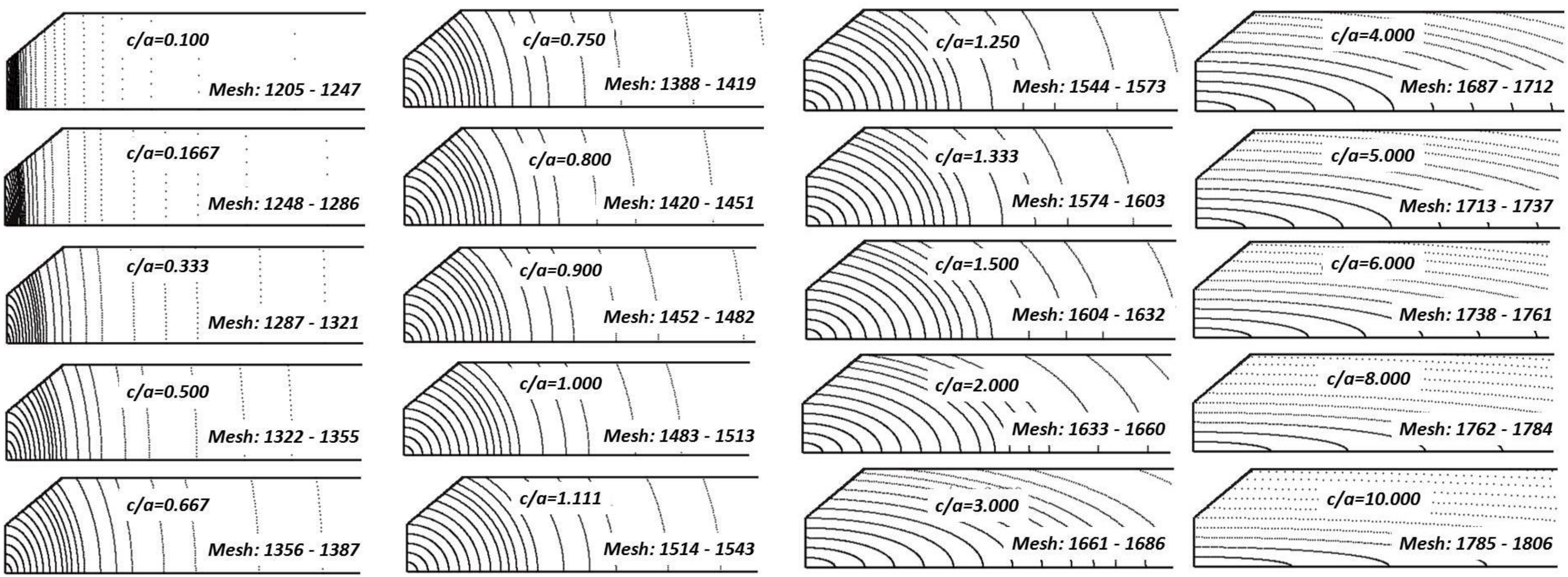


# Crack Fronts

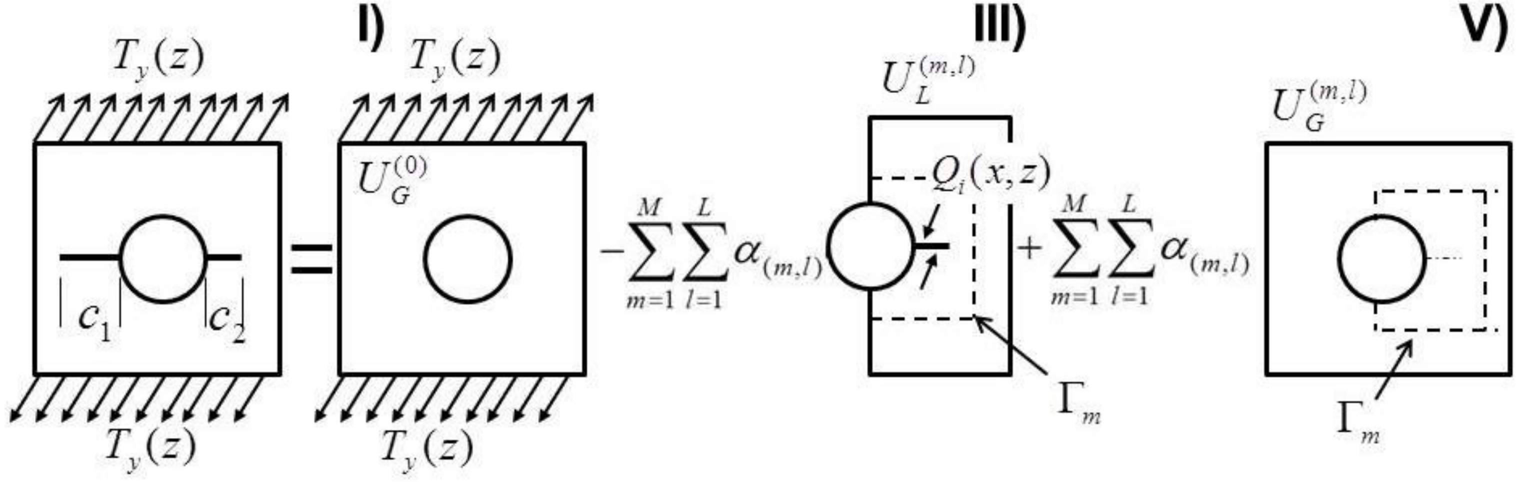
Example: 602 crack fronts for R/t and *b/t=0.50* fixed.

The number of unique twin crack scenario is $\approx 183k$.

With 14 *R/t*-values and 4 *b/t*-values there are $\approx 10.2M$ unique solutions.



# A mathematical splitting method for fast $K_I$ -analysis of multiple cracks

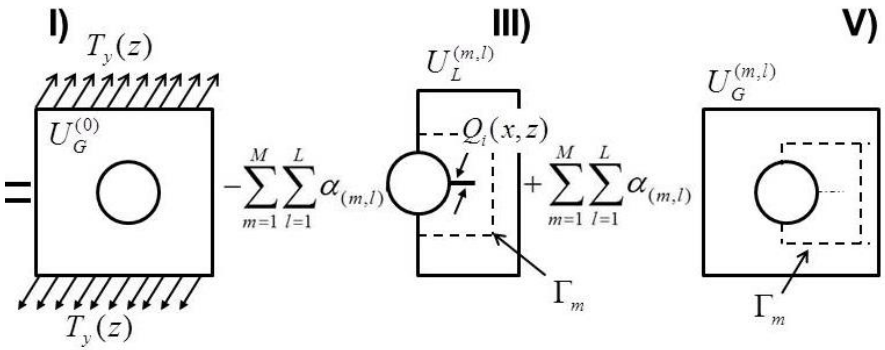



$$U = U_{global}^{(0)} + \sum_{m=1}^{M} \sum_{l=1}^{L} \alpha_{m,l} \cdot (U_{local}^{(m,l)} + U_{global}^{(m,l)})$$

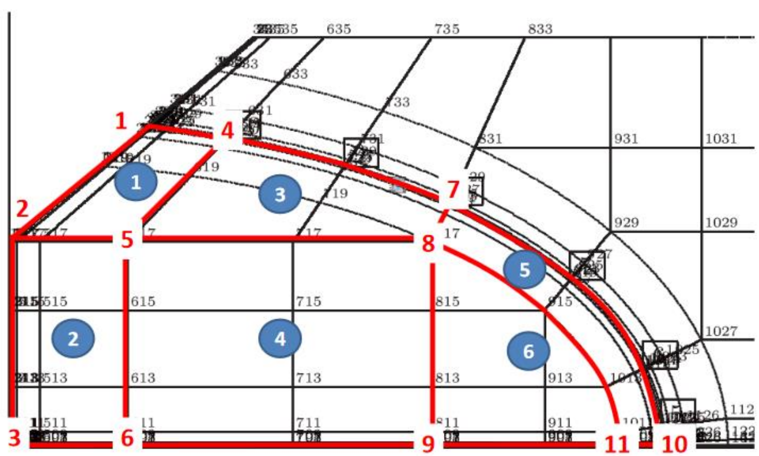
$$K_I^{(0)}(\phi), K_{II}^{(0)}(\phi), K_{III}^{(0)}(\phi) = \sum_{m=1}^{M} \sum_{l=1}^{L} \alpha_{m,l} \cdot (K_I^{(m,l)}, K_{II}^{(m,l)}, K_{III}^{(m,l)})$$

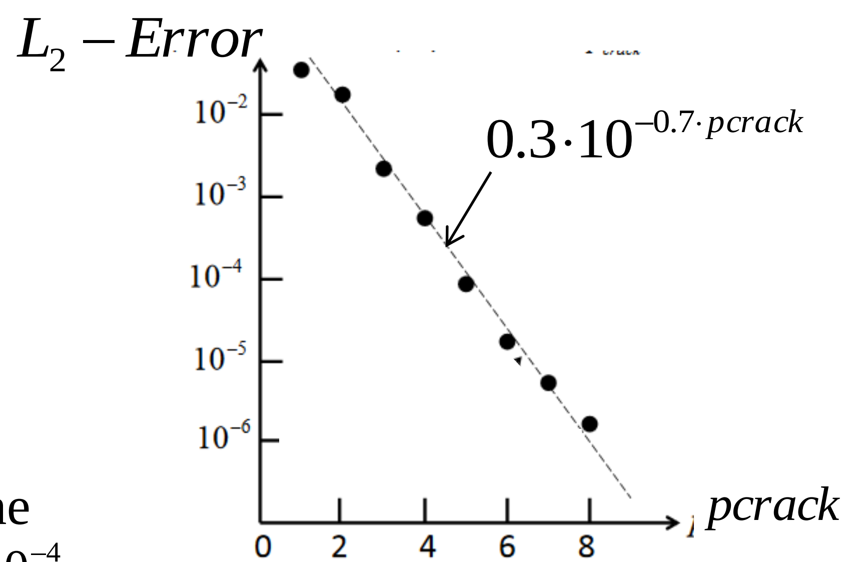
Ref: I. Babuska and B. Andersson, "The Splitting Method as a Tool for Multiple Damage Analysis", SIAM Journal Scientific Computing, Vol. 26, No 4, 2005, pp. 1114-1145.

The unknown coefficients $\alpha_{m,l}$ are determined from the condition that stresses shall be zero on the two crack faces (in the least square sense).



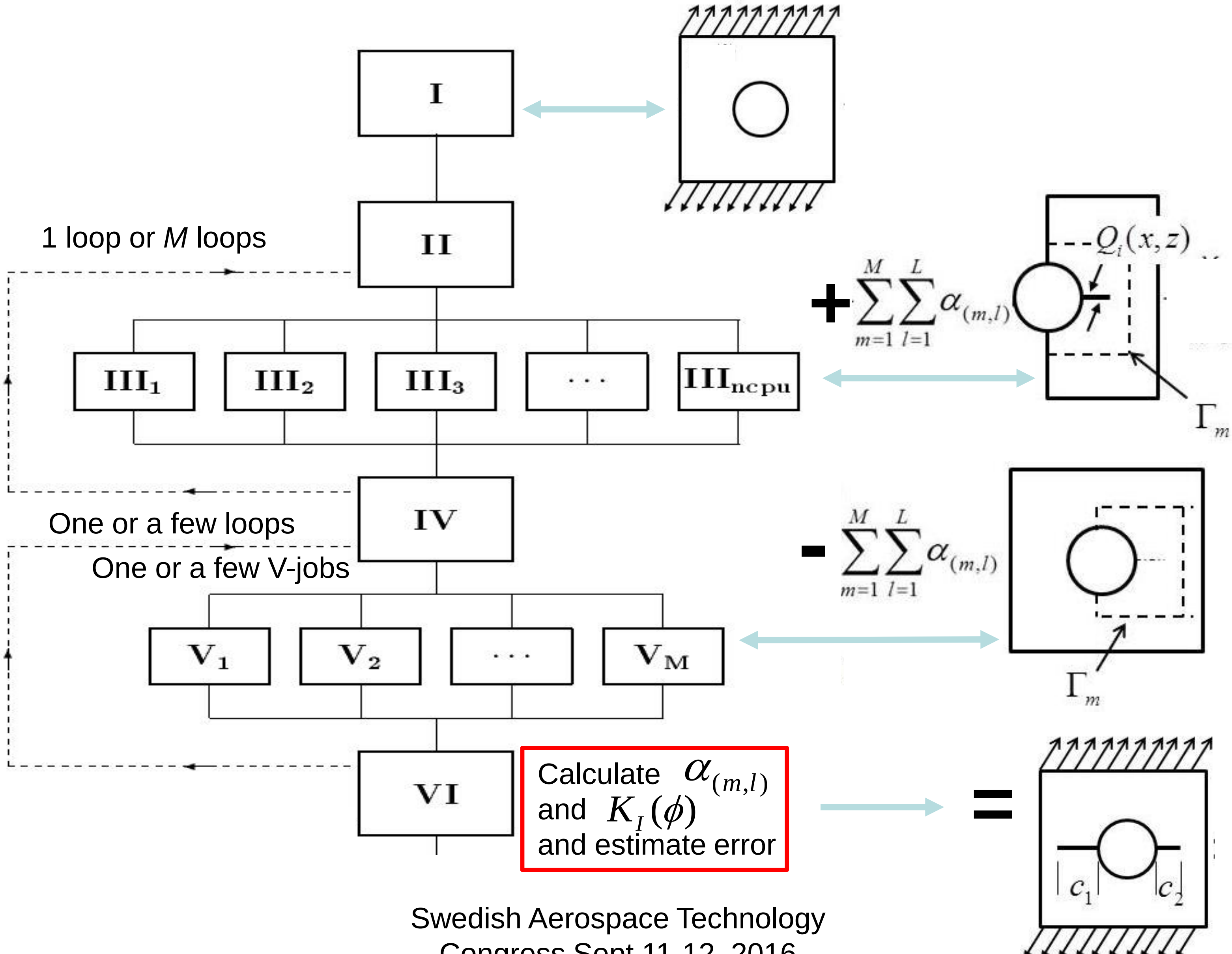

It is important to select the functions $Q_i(x,z)$, and $M$ and $L$ properly (easy to do)






Mesh on crack face used to determine *M(p)* and *L(p)* for prescribed error $10^{-4}$

# A HPC-based System for generation of large $K_I$ - Data Bases



# The Database with $K_I(\phi)$ -data

**Summary:**

- 10.2*M* twin crack scenario
- 34*k* single crack scenario
- Tension, bending and pin loading.

This gives 61.3*M* $K_I(\phi)$ - functions of the type shown in the figure to right.

The maximum relative error in each $K_I$ - function of order 0.001 or less.

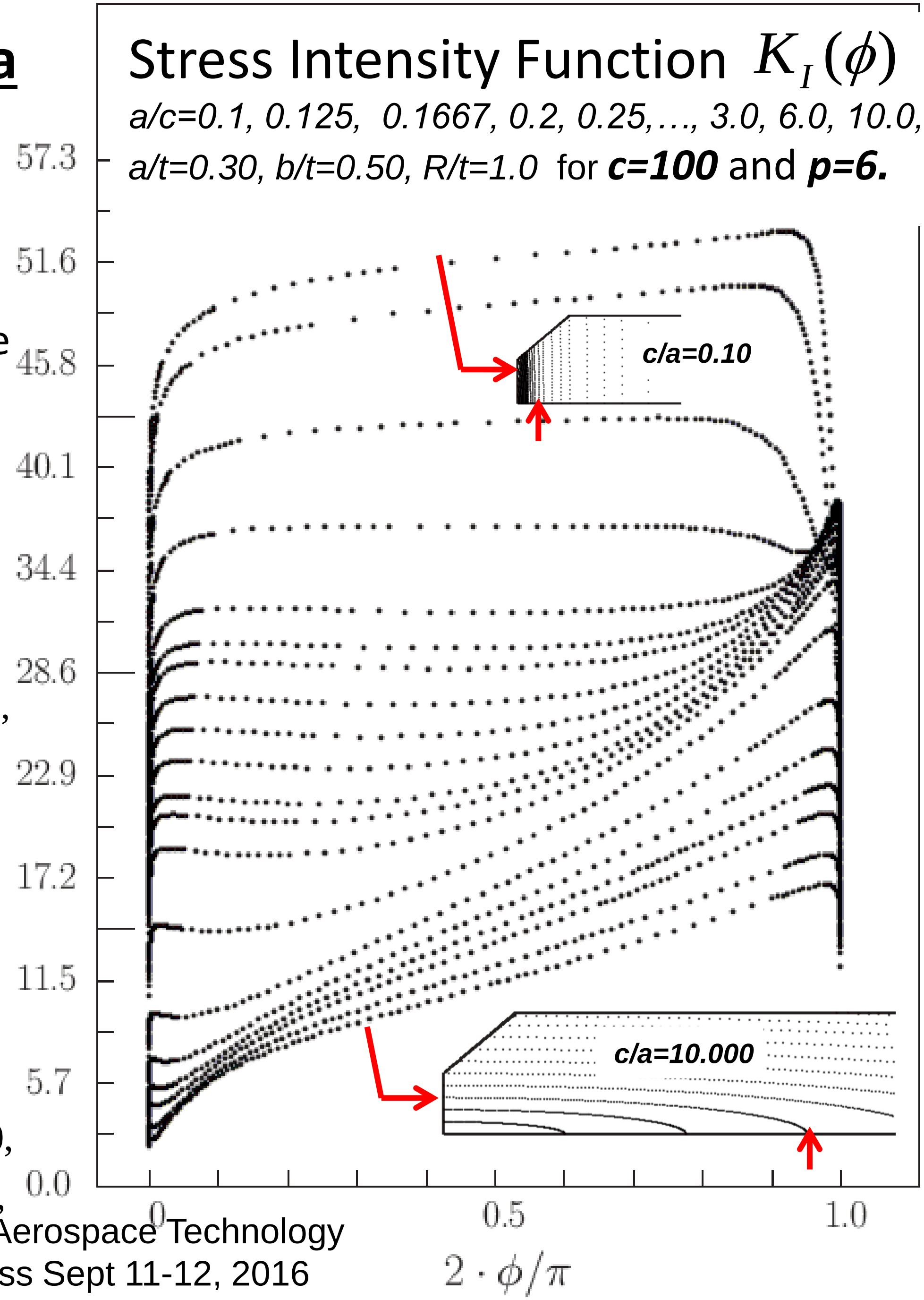
The parameter space is:

$c/a = 0.10,\ 0.1667,\ 0.333,\ 0.5000,\ 0.667,\ 0.75,\ 0.800,\ 0.900,\ 1.000,\ 1.111,\ 1.25,\ 1.333,\ 1.500,\ 2.00,\ 3.000, 4.00, 5.00, 6.00,\ 8.00,\ 10.00.$

$R/t = 0.10,\ 0.20,\ 0.50,\ 0.75,\ 1.0,\ 1.50,\ 2.0,\ 2.50,\ 3.0,\ 4.0,\ 5.0,\ 6.0,\ 8.0,\ 10.0.$

$b/t = 0.05,\ 0.25,\ 0.50,\ 0.75.$

$a/t = 0.10, 0.20, 0.30,\ 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.975, 1.05, 1.15, 1.25,\ 1.50, 1.75, 2.00, 2.50, 3.00, 4.00, 5.00,\ 6.00,\ 8.00, 10.00, \ldots, 800.00.$



# Summary

- We have shown how to derive 10.2*M* accurate twin crack scenario solutions with control of the error
- The work is based on advanced mathematical and numerical procedures as *hp*-FEM, a mathematical splitting scheme, a software STRIPE and access to HPC-resources
- The results will be delivered to users via AFGROW and possibly NASGROW.

# Acknowledgements

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