Models Based on Singular Value Decomposition for Aircraft Design

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Models for Estimating Characteristics of Components and Systems with Limited Data

- In conceptual design there is limited knowledge about subsystems, yet performance and secondary characteristics need to be estimated for estimating e.g. total weight and volume requirements of the aircraft.
- Existing methods are based on statistics of known components and systems.
- Singular Value Decomposition (SVD) is a refined method to get more information from the existing data by not only looking at relations but also look at the distribution of data in the design space.

Table of hydraulic motor data

displacement [cm3]	max power [W]	max torque [Nm]	volume [cm3]	diameter [mm]	mass [kg]	moment of inertia [kg cm2]	power intensity k_p [kW/kg]	mean pressure (torque density) p_m [bar]	power density ρ_p [W/cm3]	torque intensity k_T [Nm/kg]	Inertia fraction ε_J
59.00	44000	331.00	2203	117	8.5		5.18	1.50	19.97	38.94	NA
103.00	62000	572.00	3076	123	12.50		4.96	1.86	20.16	45.76	NA
5.00	24730	33.00	742	84	5.00	1.6	4.95	0.44	33.33	6.60	0.018
10.00	45338	66.00	1082	94	7.5	3.9	6.05	0.61	41.90	8.80	0.024
14.00	60142	96.00	1282	102	8.3	4.2	7.25	0.75	46.91	11.57	0.019
19.00	71919	127.00	1683	114	11.00	11.00	6.54	0.75	42.73	11.55	0.031
22.00	62000	140.00	1596	118	11.00	15.00	5.64	0.88	38.85	12.73	0.039
28.00	79000	178.00	1596	118	11.00	15.00	7.18	1.12	49.50	16.18	0.039
40.00	106000	255.00	2738	150	15.00	43.00	7.07	0.93	38.71	17.00	0.051
56.00	134000	357.00	2374	133	21.00	85.00	6.38	1.50	56.44	17.00	0.092
71.00	132000	509.00	5967	170	34.00	121.00	3.88	0.85	22.12	14.97	0.049
125.00	190000	895.00	9389	200	61.00	300.00	3.11	0.95	20.24	14.67	0.049
250.00	321000	1790.00	20838	265	120.00	959.00	2.68	0.86	15.40	14.92	0.046
						Average values	5.45	1.00	34.33	17.74	0.042

Multiple regression models

$$\log \widehat{m} = a_{10} + a_{11} \log P + a_{12} \log T$$

$$\log \widehat{V} = a_{20} + a_{21} \log P + a_{22} \log T$$

$$\log \widehat{J} = a_{30} + a_{31} \log P + a_{32} \log T$$

This can also be written as

$$\widehat{V}_s = 10^{a_{10}} P^{a_{21}} T^{a_{22}}$$

$$\widehat{m}_s = 10^{a_{10}} P^{a_{11}} T^{a_{12}}$$

$$\widehat{J}_s = 10^{a_{30}} P^{a_{31}} T^{a_{32}}$$

Mass and volume estimation

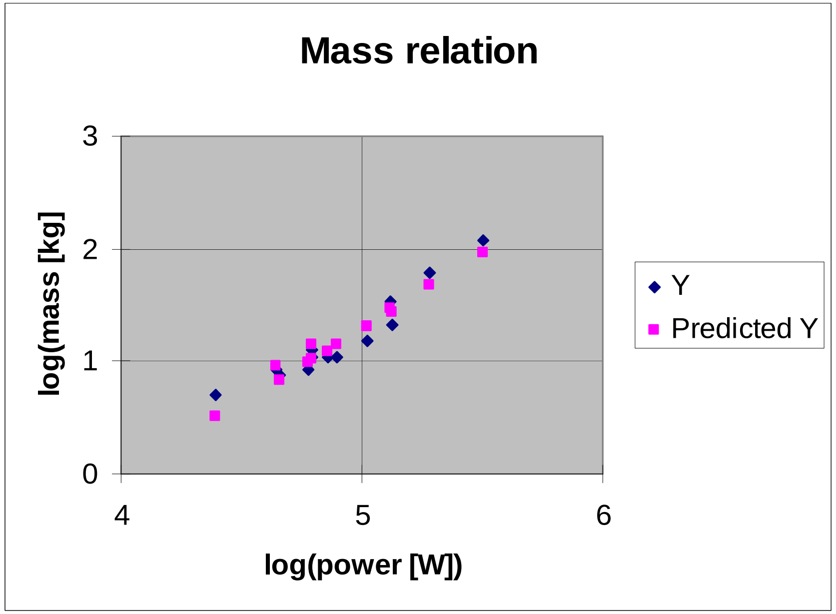
Hydraulic motor

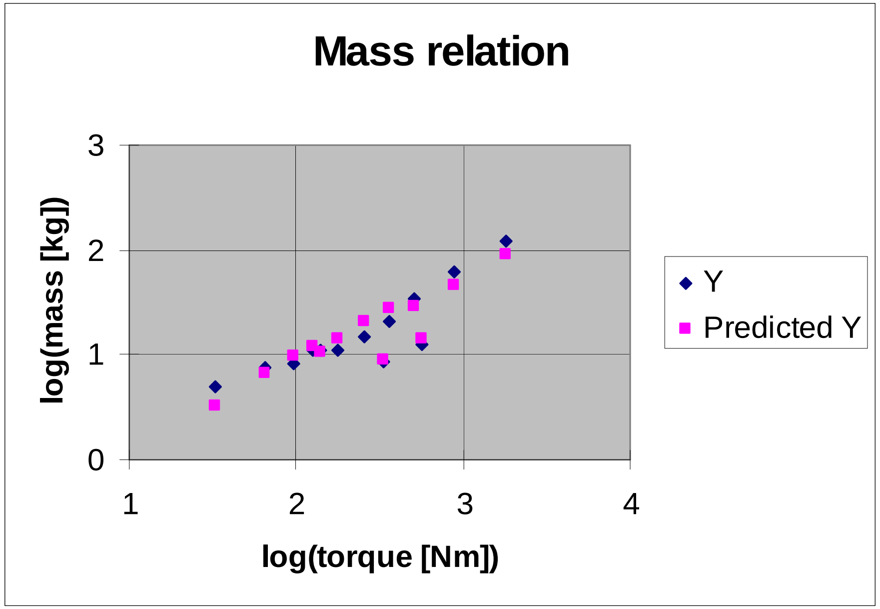
$$\widehat{m}_s = 7.26 \cdot 10^{-5} P^{0.99} T^{0.202}$$

$$\widehat{V}_s = 0.53 \cdot 10^{-6} P^{0.493} T^{0.53}$$

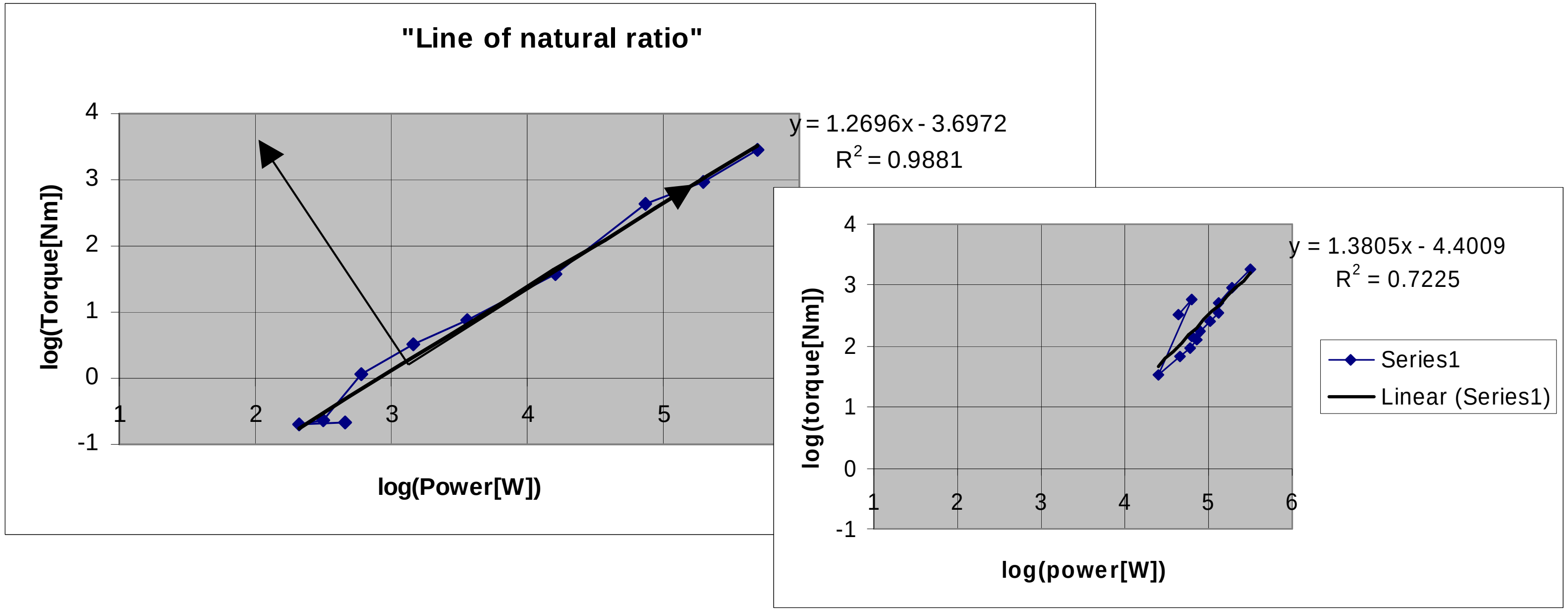
$$\widehat{J}_s = 1.34 P^{-0.734} T^{2.12}$$

Estimation model of mass for hydraulic motor

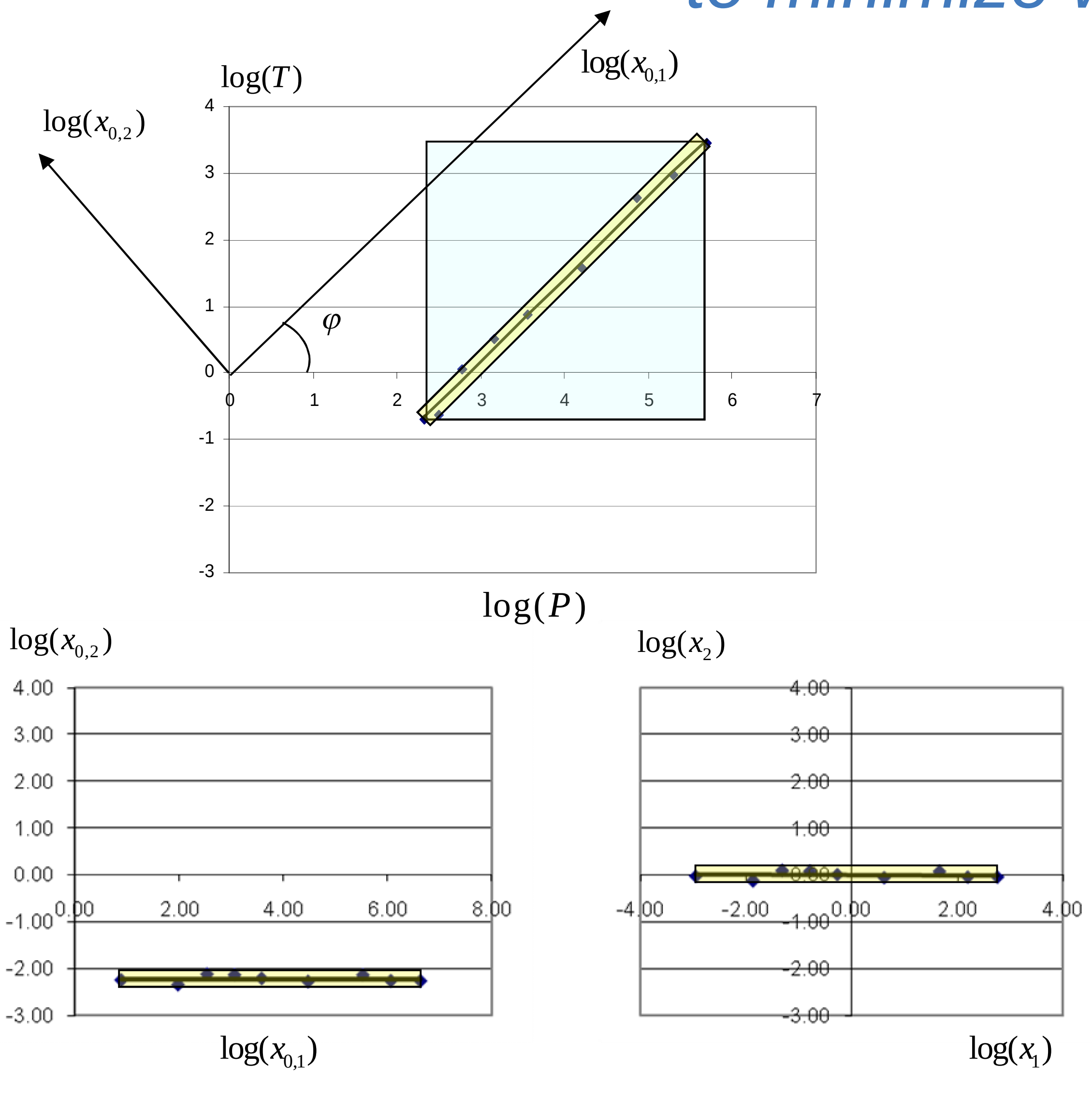




Main axis of data points (electric motor)



Prinicipal Component Analysis
to minimize waste of design space



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Singular Value Decomposition, SVD

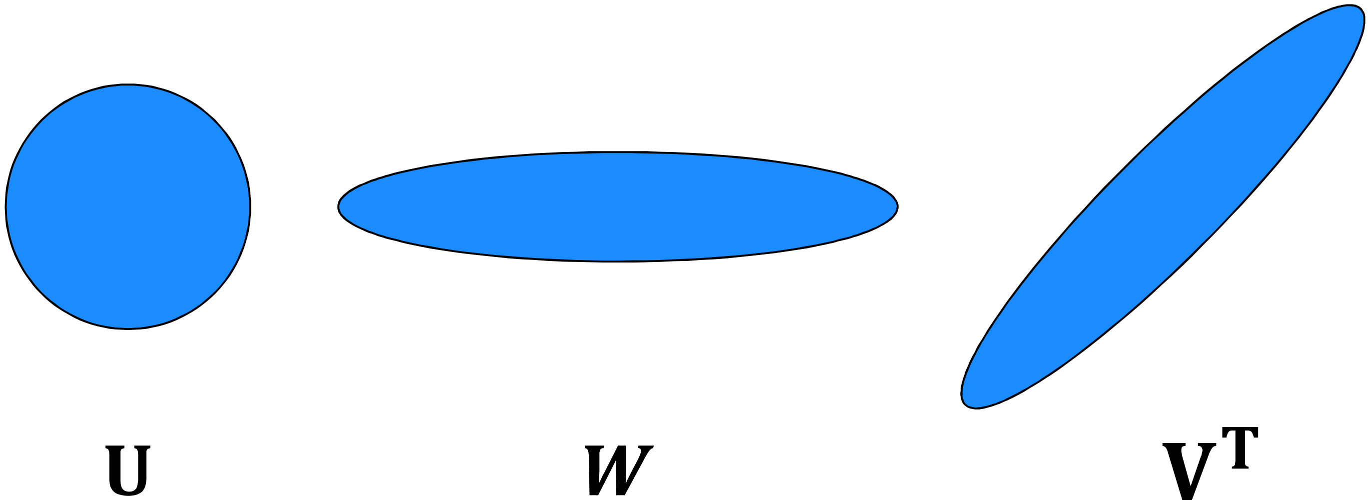
Consider the data set X which is a m n matrix. Then there exist a decomposition of the form:

$$\mathbf{X} = \mathbf{U} \times \mathbf{W} \times \mathbf{V}^{\mathbf{T}}$$

Here W is diagonal. This is called the *Singular Value Decomposition*.

$$\begin{pmatrix} x_{11} & x_{12} & x_{11} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & u_{11} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \\ u_{41} & u_{42} & u_{43} \end{pmatrix} \times \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \times \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix}^{\mathbf{T}}$$

The meaning of the matrices



$$\mathbf{X} = \mathbf{U} \times \mathbf{W} \times \mathbf{V}^{\mathrm{T}}$$

Singular Value Decomposition, SVD

To estimate parameters and properties the following equation is then used:

$$\mathbf{X} = \Psi \times \mathbf{W} \times \mathbf{V}^{\mathbf{T}}$$

.

Here Ψ is a the new independent SVD variable

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} \psi_1 & \psi_2 & \psi_3 \end{pmatrix} \times \left(\begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \times \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix}^{\mathbf{T}} \right)$$

Singular Value Decomposition, SVD

- Note that since the orthogonal parameters are sorted in ascending order it is often sufficient to use only a few input parameters. In this example it could be reduce to just one or two parameters so that the system gets reduced to:

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} \psi_1 & \psi_2 \end{pmatrix} \times \left(\begin{pmatrix} w_1 & 0 \\ 0 & w_2 \end{pmatrix} \times \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \\ v_{31} & v_{32} \end{pmatrix}^{\mathbf{T}} \right)$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} \psi_1 & \psi_2 & \psi_3 \end{pmatrix} \times \left(\begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \times \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix}^{\mathbf{T}} \right)$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} \psi_1 & \psi_2 & \psi_3 \end{pmatrix} \times \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix}$$

Singular Value Decomposition, SVD
Procedure

- Set up a dataset to establish the model
- Run the SVD analysis
- Determine how many parameters that need to be used.
- The model can e.g. be used used in optimization.

Example: Aircraft Sizeing

	pax	range km	wingarea m2	span m	length m	thrust kN	empty kg	maxTOW kg	maxFuel l	Cost MUSD
ER145LR	50	2873	51,2	20,04	29,87	164,6	22000	50790	17900	47
Bombardier CRJ-200ER	50	3045	48,35	21,21	26,77	77,66	14016	24041	6489	30
CRJ900	88	2376	70,6	24,85	36,4	129	23179	41640	12695,7	38,9
Bombardier CRJ-900ER	90	2376	70,61	24,85	36,4	118,8	21433	32999	8887	38,93
ER195	106	4260	92,5	28,7	38,6	164,6	28970	50790	12971	47
A320	164	6100	122,6	35,8	37,57	240	42600	78000	30190	97
B737800	177	4440	105,4	35,8	37	242	44700	85100	26020	93,3
B767300ER	269	11090	283	47,6	54,9	552	900010	186880	91400	15,8
A330300	335	11300	439,4	60,3	63,7	632	124500	242000	139090	253,7
A340500	359	16060	361,6	65,5	67,9	1040	170500	372000	195880	261,8
B777ER	400	13600	436	64,8	73,9	1024	167000	351000	181283	320,2
B747400	565	13450	525	64,6	70,6	1104	184600	396890	216840	260
A380800	644	10400	845	79,75	72,73	1360	252200	590000	323546	260

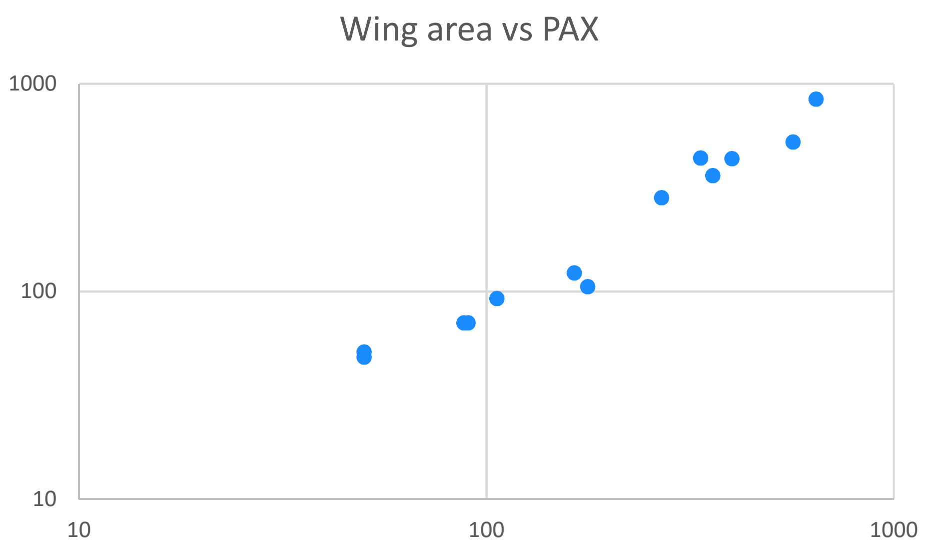
Using logarithm of values and centering the data.

$$x'_{ij} = \log(x_{ij}) - \frac{1}{n}\sum_{i=1}^{n}\log(x_{ij})$$

When an estimate in these coordinates have been found it has to be retransformed to yeld the actual value of estimate

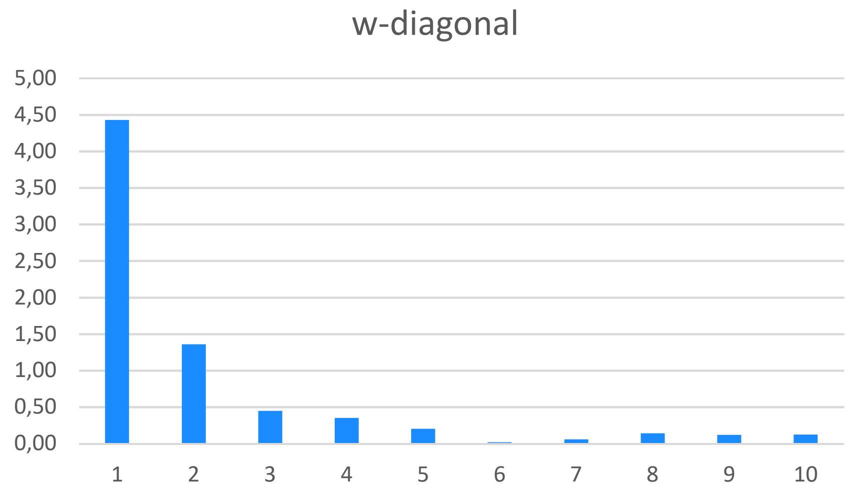
$$\hat{x}_j = 10^{\hat{x}'_j + \frac{1}{n}\sum_{i=1}^{n}\log(x_{ij})}$$

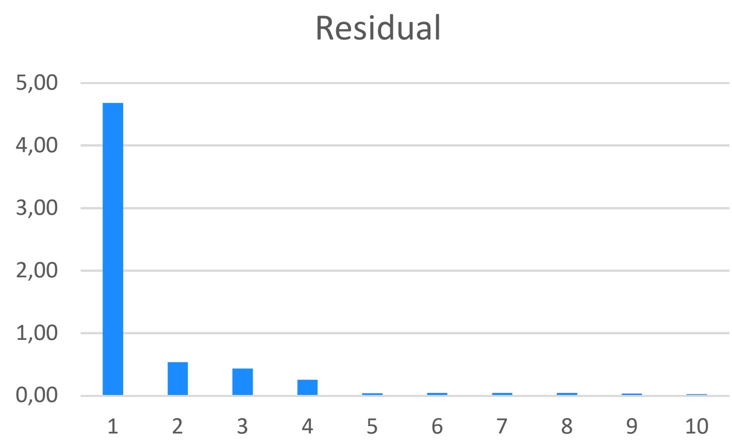
Correlation



SVD model of Civil aircraft

	Rel error	ER145LR	Estimate	Adjusted	Result	Average											SVD variables	w-diagonal	residual
pax	0.00	50.00	50.00	1.70	-0.57	2.27	0.355	-0.010	0.077	-0.009	0.029	-0.004	0.008	-0.008	0.000	-0.001	-1.00	4.36	2.69
range km	0.00	2873.00	2872.97	3.46	-0.34	3.80	0.288	0.018	-0.032	-0.082	-0.010	0.002	0.011	-0.003	0.001	0.000	-0.19	1.34	0.63
wingarea	0.00	51.20	51.20	1.71	-0.53	2.24	0.407	-0.003	0.055	0.008	-0.041	-0.002	0.002	0.002	0.000	0.005	-2.62	0.42	0.64
span m	0.00	20.04	20.04	1.30	-0.30	1.60	0.200	-0.011	0.025	-0.014	-0.002	-0.002	0.000	0.010	-0.004	-0.009	1.59	0.35	0.35
length m	0.00	29.87	29.87	1.48	-0.19	1.67	0.149	-0.004	0.015	-0.001	-0.010	0.027	-0.014	-0.006	0.002	-0.006	-0.02	0.21	0.03
thrust kN	0.00	164.60	164.60	2.22	-0.32	2.54	0.414	-0.011	-0.023	0.017	0.012	0.021	0.007	0.000	-0.004	0.005	-0.13	0.14	0.03
empty kg	0.00	22000.00	21999.76	4.34	-0.51	4.85	0.461	0.242	-0.004	-0.011	0.013	-0.005	-0.017	0.004	0.000	0.003	-0.07	0.12	0.03
maxTOW l	0.00	50790.00	50789.39	4.71	-0.36	5.07	0.453	-0.016	-0.019	0.027	0.007	0.005	0.014	0.009	0.004	-0.002	-0.17	0.08	0.03
maxFuel l	0.00	17900.00	17899.81	4.25	-0.41	4.66	0.576	-0.013	-0.044	0.031	-0.010	-0.016	-0.001	-0.010	-0.001	-0.004	0.16	0.03	0.03
Cost MUSI	0.00	47.00	47.00	1.67	-0.28	1.95	0.321	-0.281	-0.006	-0.018	0.009	-0.005	-0.016	0.003	0.001	0.003	1.11	0.05	0.02
	0.00																		





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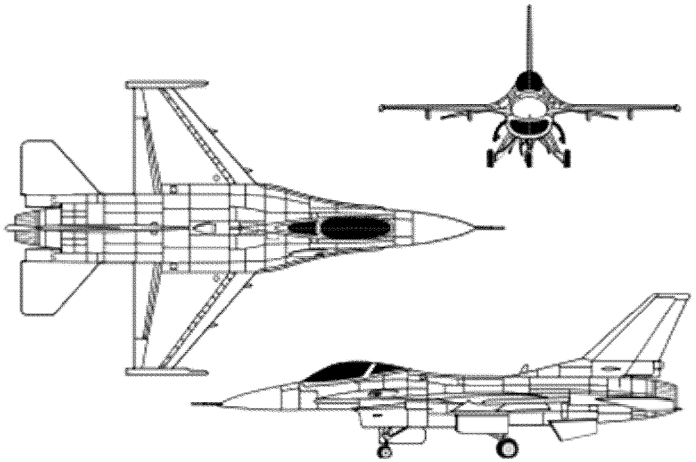
Modelling of Military Aircraft. Estimation With Limited Data

Name	Service ceiling [m]	Max speed (Mach)	empty [kg]	maxTOW [kg]	Range/max Fuel	Max thrust [kN]	wingarea [m^2]	span [m]	length [m]	Stealth (1=no, 2=yes)
Typhoon	19000	2.00	10000	21000	0.925	180.0	51.2	10.5	15.96	1
Rafale C	19810	2.00	9060	15060	0.5129108	174.0	46	10.9	15.3	1
PAK FA	20000	2.30	18000	35000	0.3398058	334.0	78.8	13.95	19.8	2
BAE Hawk 200	15250	0.84	4128	9101	0.6558824	26.0	16.69	9.39	11.38	1
F-5E	15800	1.45	4349	11214	0.4385486	44.4	17.28	8.13	14.5	1
L-159	13200	0.76	4350	8000	1.0122502	28.2	18.8	9.54	12.72	1
M-346	14716	0.86	4610	9500	0.9905	28.0	23.52	9.72	11.49	1
Mitsubishi F-2A	18000	2.00	9527	22100	0.137139	131.0	34.84	11.13	15.52	1
KAI T-50	14630	1.50	6470	12300	0.8609302	78.7	23.69	9.45	13.4	1
Atlas Cheetah C	17000	2.20	6600	13700	0.3794561	71.0	35	8.22	15.55	1
Mirage 2000	17060	2.20	7500	17000	0.3381307	95.1	41	9.13	14.36	1
F-15C	19810	2.50	12975	30845	0.7471483	212.0	56.48	13.05	19.43	1
Mig 29	18013	2.40	10900	20000	0.4085714	184.4	38	11.36	17.32	1
Su 27S	19000	2.35	16380	23140	0.2978723	245.2	62.04	14.7	21.94	1
J-10	18000	2.20	9750	19277	0.2444444	130.0	39	9.75	15.49	1
JA 37	18000	2.10	9500	20000	0.5	125.0	46	10.6	16.4	1
J 35F	18000	2.20	7425	11914	0.5580357	78.4	49.22	9.42	15.35	1
MIG-31	20600	2.83	21820	46200	0.4590253	304.0	61.6	22.69	22.69	1
F-35A	18288	1.70	13199	31800	0.2648533	191.0	42.7	10.7	15.4	2
F-22	20000	2.25	19700	38000	0.2073171	312.0	78	13.56	18.9	2
JF-17	16920	1.80	6586	12500	0.8668085	84.6	24.4	9.45	14.93	1
Gripen C	15240	2.00	6620	12700	0.7048458	80.0	30	8.4	14.1	1
F-16C Block 50	15240	2.00	8495	19200	0.4214521	127.0	27.88	9.45	15.03	1



Modelling of Military Aircraft. Estimation With Limited Data Example: F-16

	Rel error	F16 block 50	Estimate	Adjusted	Result	Average											SVD variables
Service ceiling [m]	0.13	15240	17190	4.24	-0.01	4.24	-0.047	0.004	-0.006	0.001	-0.004	0.001	-0.006	-0.007	0.016	0.005	0.04
Max speed (Mach)	0.07	2.00	2.14	0.33	0.07	0.26	-0.129	0.007	-0.078	0.005	0.006	0.025	0.014	0.001	-0.001	0.007	-0.20
empty [kg]	0.07	8495	7922	3.90	-0.05	3.95	-0.206	0.020	0.025	-0.014	-0.010	-0.006	0.009	-0.007	-0.012	0.002	-0.83
maxTOW [kg]	0.09	19200	17407	4.24	-0.01	4.25	-0.200	0.005	0.038	-0.025	0.011	0.036	-0.014	0.001	0.000	-0.003	-0.37
Range/maxFuel	0.07	0.42	0.45	-0.35	-0.02	-0.32	0.141	0.184	0.008	0.007	0.007	0.006	0.001	0.000	0.000	0.000	2.00
Max thrust [kN]	0.00	127	127.00	2.10	0.06	2.05	-0.325	0.039	-0.016	0.010	0.035	-0.025	-0.006	0.002	0.002	-0.001	0.00
wingarea [m^2]	0.00	27.88	27.88	1.45	-0.13	1.58	-0.181	0.030	-0.006	0.038	-0.054	0.002	-0.008	0.002	0.000	-0.002	0.00
span [m]	0.00	9.45	9.45	0.98	-0.06	1.03	-0.074	0.026	0.029	-0.047	-0.020	-0.009	0.009	0.005	0.005	0.009	0.00
length [m]	0.03	15.03	15.48	1.19	-0.01	1.20	-0.069	0.013	-0.007	-0.016	-0.007	0.000	0.019	0.000	0.007	-0.015	0.00
Stealth (1=no, 2=ye	0.00	1	1.00	0.00	-0.04	0.04	-0.053	-0.021	0.067	0.049	0.015	0.008	0.016	0.001	0.003	0.003	0.00
	0.13																

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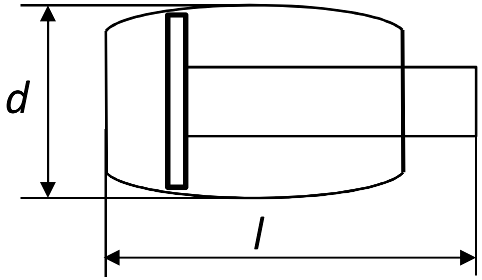
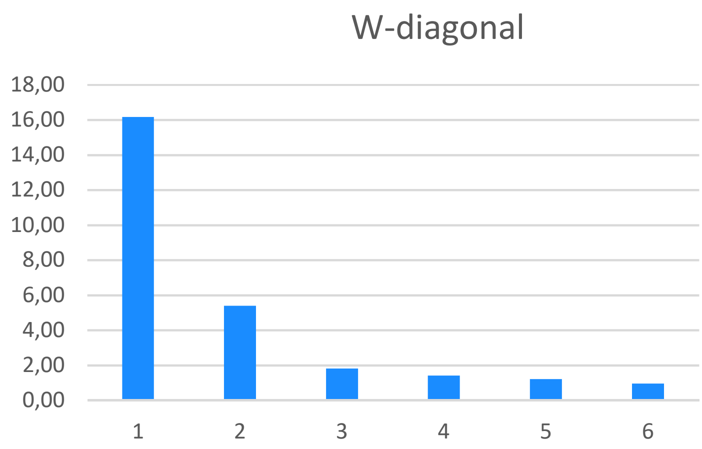
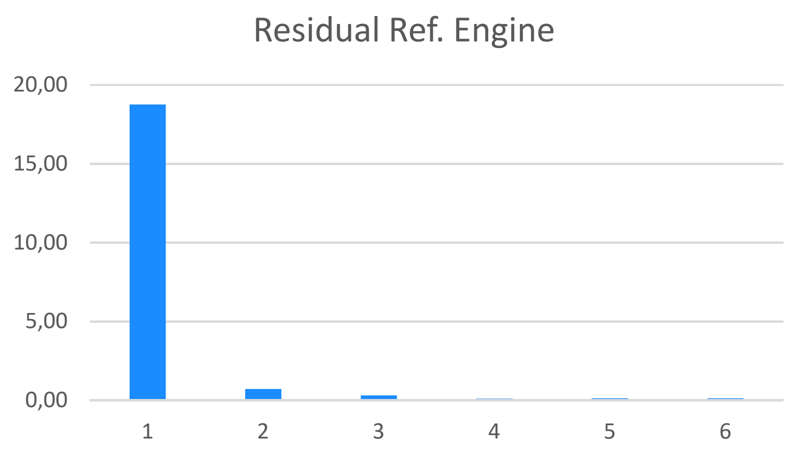


SVD model of Aero Engine

	Rel error	TF1000	Estimate	Adjusted	Result	Average							SVD variables	w-diagonal	residual
Bpr+1	0.00	9.14	9.14	0.96	0.29	0.67	-0.106	0.208	-0.011	0.028	-0.011	0.001	2.34	16.17	18.75
T [kn]	0.00	4.45	4.45	0.65	-1.30	1.94	-0.483	-0.025	0.025	0.008	0.000	-0.029	2.40	5.38	0.71
Sfc [1/hr]	0.00	0.40	0.40	-0.40	-0.06	-0.34	0.074	-0.101	0.011	0.056	0.000	0.008	-2.33	1.81	0.31
w [kg]	0.00	129.56	129.56	2.11	-1.13	3.24	-0.463	-0.042	-0.004	-0.008	-0.023	0.026	0.41	1.41	0.11
d [m]	0.00	0.58	0.58	-0.23	-0.42	0.18	-0.213	0.042	0.016	0.000	0.046	0.016	-0.23	1.20	0.12
l [m]	0.00	1.42	1.42	0.15	-0.31	0.46	-0.158	-0.042	-0.074	0.007	0.013	-0.007	1.66	0.96	0.12
	0.00														





Singular Value Decomposition, SVD

- The x vector can contain both parameters and characteristics, that is; both input and output variables.
- The new SVD parameter set has no direct physical meaning, but the prinicipal parameter can usually be regarded as a ”size” variable.
- The SVD parameters are aligned with the space of the sample datapoints, meaning that explicit constraints can be put on them when optimizing.
- A good parametrisation is usually close to the SVD parameter set.

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