Models Based on Singular Value Decomposition for Aircraft Design

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Abstract
Models based on statistics have a long history in aircraft design. They are used for both components, subsystems and aircraft as a whole. The idea is that existing designs represent a knowledge base of what are achievable also for a new designs. In this way characteristics of a new design with similar characteristics can be estimated with some accuracy without going into details about the design as such.

With conventional statistical methods, e.g. multiple regression analysis, some entities are assumed to be independent variables, while other are assumed to be dependent from these. The fact is that there usually are rather strong correlations among all the entities. An alternative very useful technique for this kind of models is the Singular Value Decomposition SVD. It is a technique where a set of synthetic orthogonal parameters are generated. These are automatically arranged to be truly independent and have the attractive property that they will have an influence of rapidly descending order. This means that only a few parameters can be used to represent what appears to be complex relations. In this paper this is demonstrated on datasets of civil and military aircraft, as well as for components and subsystems.

Keywords
Abstract; Statistical models, conceptual design, surrogate models.

INTRODUCTION

Models based on statistics have a long history in aircraft design, most importantly for weight estimation, see e.g. Ahl (1969), St. John (1969). They have a broad applications in aircraft design as described e.g. in Torenbeek (1980), Roskam (1985). They are used for both components, as in Krus (2005), subsystems and aircraft as a whole. The idea is that existing designs represent a knowledge base of what are achievable also for a new designs. In this way characteristics of a new design with similar characteristics can be estimated with some accuracy without going into details about the design as such.

Using singular value decomposition, introduced in Mandel (1982), it is possible to create a model that has a few synthetic parameters as inputs and all the attribute of the design as outputs. This includes both design parameters and functional characteristics. It is then possible to quickly estimate a design from given requirements, by solving the resulting system of equations. Interestingly it can also be used to estimate performance and other characteristics from limited data.

Another very useful application is for modelling of components and subsystems. In the paper an engine model is presented that with high accuracy can relate engine dimension, i.e. diameter, length, weight, bypass ratio, trust and specific fuel consumption. It is also possible to include year of introduction as one variable and in this way also have a mechanism for technology evolution over time.

In a design situation the SVD model can be used in the role of a meta model. Instead of making a parametric design of a higher fidelity that is optimized for each situation, It is possible to optimize for a few situations and then build a SVD-model based on these. In this way a meta model with high accuracy can be obtained. Ones an optimal solution has been reached it can be recalculated and be added to the set of data points the SVD model is based on.

Finally, SVD analysis can be used of test a given parametrization by studying the correlation with the ideal SVD parameter set. This is useful since it sometimes an advantage to have a parametrization that have a clearer interpretation than the synthetic SVD parameter set can provide. Interestingly, it is also possible to derive the number of driving requirement in a design by studying a number of instances of a particular kind of product.
Singular Value Decomposition (SVD) is a technique that is related to principle component analysis. The result is essentially the same but it involves an elegant mathematical method to obtain a model that is aligned with the main axis of the data set. Consider the data set X which is a matrix. Then there exist a decomposition of the form:

$$\mathbf{X} = \mathbf{U} \times \mathbf{W} \times \mathbf{V}^{\mathbf{T}} \qquad (1)$$

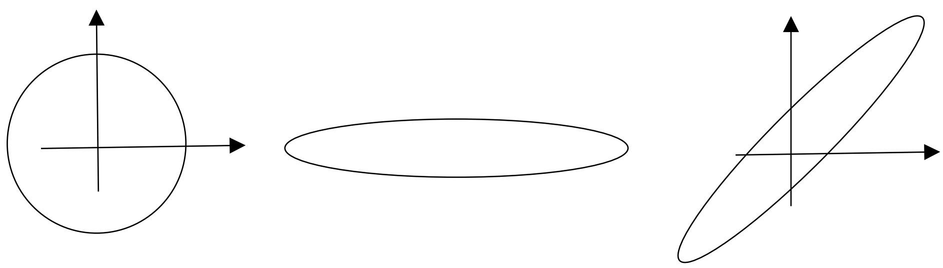
where W is diagonal. This is the Singular Value Decomposition, SVD. This can look like this:

$$\begin{pmatrix} x_{11} & x_{12} & x_{11} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & u_{11} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \\ u_{41} & u_{42} & u_{43} \end{pmatrix} \times \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \times \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix}^{\mathbf{T}} \qquad (2)$$

The consequence of this operation is that if each row in the X and U matrix represents a data set of the entity that should be modelled., any point in U is mapped onto X trough the matrix product . Usually the resulting matrices are arranged in such a way the diagonal elements of the W-matrix are in descending order. Hence the influence of the u variables are in descending order in each row, which means that the last ones can be omitted in order to get a simpler model without too much loss in accuracy. However, for this to be valid the dataset should first be centred around the mean value. This can be done by subtracting the average of each column in the x-vector from the values of each column. An interesting property of the U matrix is then that the sum of the variance of each column is one. That is:

$$\sum_{i=1}^{n} u_{ij}^2 = 1 \qquad (3)$$

This means that all columns (that is parameters) have the same deviation . The matrix is then a weight matrix with only diagonal elements, and is a matrix that rotates the coordinate system from the main axis into .



$\mathbf{S}$ $\qquad\qquad$ $\mathbf{S} \times \mathbf{W}$ $\qquad\qquad$ $\mathbf{X} = \mathbf{S} \times \mathbf{W} \times \mathbf{V}^{\mathbf{T}}$

To estimate parameters and properties the following equation is used:

$$\mathbf{X} = \mathbf{S} \times \mathbf{W} \times \mathbf{V}^{\mathbf{T}} \tag{4}$$

Here the $\mathbf{X}$ and $\mathbf{S}$ are vectors. $\mathbf{S}$ is the input vector with *SVD-parameters* that are orthogonal and $\mathbf{X}$ is the estimated values of parameters and properties. This can look like this:

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} s_1 & s_2 & s_3 \end{pmatrix} \times \left(\begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \times \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix}^{\mathbf{T}} \right) \tag{5}$$

With

$$\begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix}^{\mathbf{T}} = \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \times \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix}^{\mathbf{T}} \tag{6}$$

This can be written as

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} s_1 & s_2 & s_3 \end{pmatrix} \times \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix}^{\mathbf{T}} \tag{7}$$

Or

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \times \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} \tag{6}$$

Note that since the orthogonal parameters are sorted in ascending order it is often sufficient to use only a few input parameters. In this example it could be reduced to just one or two parameters so that the system gets reduced to:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{pmatrix} \times \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} \tag{7}$$

Note that the individual element in the k matrix is the same as before. The same result is therefore achieved by just setting the last element in the input vector in equation (5) to zero.

One issue with this model is that the elements variance of the S-vector is dependent on the number of data set, and hence also the K-matrix. Therefore, it can be suitable to normalise the S-vector so that the variance of the elements is one. This is simply done by dividing each element with the deviation and then consequently multiplying the elements in the K-matrix with the same value.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} k_{0,11} & k_{0,12} \\ k_{0,21} & k_{0,22} \\ k_{0,31} & k_{0,32} \end{pmatrix} \times \begin{pmatrix} s_{0,1} \\ s_{0,2} \end{pmatrix} \tag{8}$$

Where

$$s_{0,j} = \frac{s_j}{\sigma_S} = s_j \sqrt{n} \tag{8}$$

And consequently

$$k_{0,ij} = \sigma_S k_{ij} = \frac{k_{ij}}{\sqrt{n}} \tag{9}$$

Example: Transport aircraft

As an example the characteristics for transport aircraft is used. The table below some aircraft with some of their characteristics. Note that there are both functional characteristic, such as range and number of passengers as well as design parameters such as wing area, wingspan and thrust.

Table 1. Civil aircraft

Name	PAX	range [km]	empty [kg]	maxTOW [kg]	maxFuel [l]	thrust [kN]	wing area [m^2]	span [m]	length [m]	Cost [M USD]
ER145LR	50	2873	11440	22000	6484	67	51.2	20.04	29.87	47
CRJ-200ER	50	3045	14016	24041	6489	78	48.35	21.21	26.77	30
CRJ-900ER	90	2376	21433	37421	8887	119	70.61	24.85	36.4	38.93
ERJ195	106	4260	28970	50790	12971	165	92.5	28.7	38.6	47
B737600	130	4440	36400	65500	26020	174	105.4	35.8	37	93.3
A320	164	6100	42600	78000	30190	240	122.6	35.8	37.57	97
B767300ER	269	11090	90010	186880	91400	552	283	47.6	54.9	185.8
A330300	335	11300	124500	242000	139090	632	361.6	60.3	63.7	253.7
A340500	359	16060	170500	372000	215260	1040	439.4	65.5	67.9	261.8
B777ER	400	13600	167000	351000	181283	1024	436	64.8	73.9	320.2
B747400	565	13450	184600	396890	216840	1104	525	64.6	70.6	260
A380800	644	10400	252200	590000	323546	1360	845	79.75	72.73	260

In order to have a better model structure the SVD analysis is made on the logarithm of the original data. The data is also centred around the mean value.

$$x'_{ij} = \log_2(x_{ij}) - \frac{1}{n}\sum_{i=1}^{n}\log_2(x_{ij}) \tag{10}$$

The result is a model of the form

$$\begin{pmatrix} \hat{x}'_1 \\ \hat{x}'_2 \\ \hat{x}'_3 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \times \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} \tag{11}$$

Where s is the SVD vector. The result from the model is then manipulated to get back to the original domain.

$$\hat{x}_j = 2^{\hat{x}'_j + \frac{1}{n}\sum_{i=1}^{n}\log(x_{ij})} \tag{12}$$

The selection of base for the log is somewhat arbitrary. Using two means that the interval -1 to 1 corresponds to a difference of a factor four. This is here considered as a reasonable compromise that mean that the left column in the K-matrix is around one. It does, however, have any other consequence. The table below shows the result from an SVD analysis. The K-matrix has been normalized, so the deviation of the SVD parameters of the datasets should be one. Here the values of the transformation matrix K are colour coded to highlight the significant values. Apparently the first column is totally dominant.

Table 2. SVD model of civil aircraft.

	Rel error	ER145LR	Estimate	Adjustec	Result	Average	K-matrix										SVD varial	w-diagona	residual
PAX	0.00	50.00	50.00	1.70	-0.56	2.26	0.357	-0.013	0.040	-0.005	0.031	-0.014	0.000	-0.003	0.002	0.000	0.000	-1.45	4.43	6.37
range [km]	0.00	2873.00	2872.97	3.46	-0.37	3.83	0.262	0.030	-0.079	-0.022	0.002	-0.008	-0.003	-0.006	0.000	0.000	0.000	-0.77	1.34	0.48
empty [kg]	0.00	11440.00	11439.88	4.06	-0.72	4.77	0.432	0.039	0.014	-0.009	0.006	0.014	0.002	-0.006	-0.003	0.002	-0.001	-0.61	0.37	0.14
maxTOW [kg]	0.00	22000.00	21999.76	4.34	-0.73	5.07	0.465	0.060	0.012	-0.002	-0.005	0.005	-0.002	0.004	0.000	-0.003	-0.001	1.12	0.20	0.14
maxFuel [l]	0.00	6483.75	6483.69	3.81	-0.87	4.68	0.591	0.083	-0.026	0.032	-0.001	0.003	0.000	0.002	0.004	0.001	0.000	-1.67	0.16	0.12
thrust [kN]	0.00	66.60	66.60	1.82	-0.69	2.51	0.444	0.002	0.010	-0.032	-0.002	0.005	-0.004	0.009	0.001	0.001	0.001	-1.68	0.11	0.07
wingarea [m^2]	0.00	51.20	51.20	1.71	-0.57	2.28	0.388	0.078	0.020	0.011	-0.019	-0.017	-0.004	-0.001	-0.004	0.000	0.000	0.22	0.09	0.02
span [m]	0.00	20.04	20.04	1.30	-0.31	1.61	0.193	0.022	0.003	0.010	0.003	0.011	0.002	-0.009	-0.001	-0.002	0.001	0.34	0.07	0.02
length [m]	0.00	29.87	29.87	1.48	-0.20	1.68	0.147	0.010	0.006	-0.012	-0.014	-0.004	0.022	-0.001	0.002	0.000	0.000	1.03	0.03	0.01
Cost [M USD]	0.00	47.00	47.00	1.67	-0.28	1.96	0.391	-0.342	-0.010	0.009	-0.003	0.000	0.001	0.001	-0.001	0.000	0.000	-0.90	0.02	0.01
year	0.00	97.00	97.00	1.99	0.03	1.95	0.006	-0.050	0.030	-0.011	-0.021	0.000	-0.008	-0.010	0.005	0.000	0.000	-0.07	0.01	0.00
	0.00																			

This is a model where the SVD-parameters can be used to define an aircraft. It is a model that can be used e.g. to optimize an aircraft for a specific kind of mission, e.g. payload and range.
Looking at the diagonal element in the w- matrix the relative importance of the SVD-parameters can be studied.

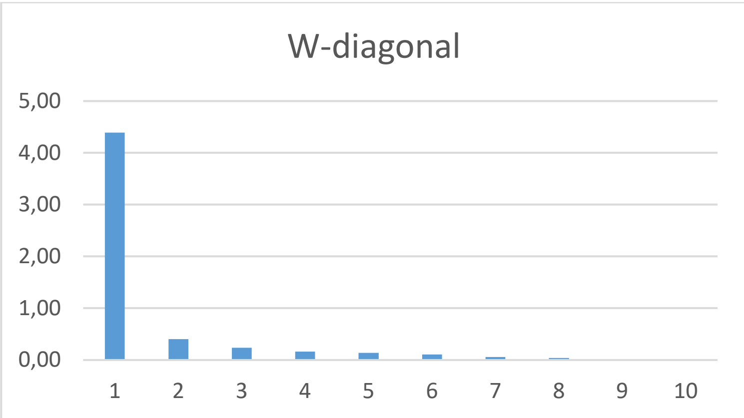


Figure 1. The influence of the SVD parameters.

It is remarkable how quickly the importance of SVD-parameters tapers off. Here already the second element have less than 10% of the first element. This means that even only one parameter can give a rather good estimate of the characteristics. There is also a column of residuals. This column represents the maximum relative error for any attribute when sequentially setting the SVD- variables to zero starting from the bottom, for this particular example (Embraer ER-145 LR). Hence, with two parameters all attributes of the aircraft will be modelled with an accuracy of less than 15%, and with more variables the influence of other parameters is probably within the accuracy of the data. It should be noted that the largest aircraft is one order of magnitude larger than the smallest (the ER-145) and that this ER-145 represents an extreme.

Looking at the transfer matrix, K, it can be see that the greatest influence of the second SVD parameter is on passengers and range. This implies that the two requirements driving the design is these two. Considering that other requirements are more or less the same for all aircraft the consistency is not to surprising, since they have been optimised towards more or less the same objectives.

MODELLING OF CIVIL AERO ENGINES.

One example of a component that is very useful to have accurate models of also at the conceptual design stage is the engine. A database of more than 400 civil aero engines was assembled from datasheets and various sources. The result is a model that can predict the relation between geometrical dimension, diameter and length, bypass ratio, mass, and thrust and specific fuel consumption.

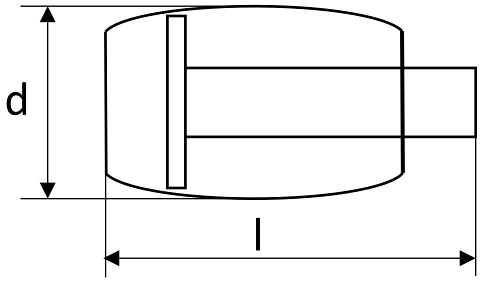


Figure 2. Civil aero Engine.

Table 3. SVD model for civil aircraft engines.

	Estimate	Adjusted	Result	Average							SVD variables	w-diagonal
Bpr+1	6.10	0.79	0.12	0.67	0.106	-0.208	0.011	-0.028	0.011	0.001	1.11	16.17
T [kn]	305.15	2.48	0.54	1.94	0.483	0.025	-0.025	-0.008	0.000	-0.029	0.02	5.38
Sfc [1/hr]	0.40	-0.40	-0.06	-0.34	-0.074	0.101	-0.011	-0.056	0.000	0.008	-0.32	1.81
w [kg]	5636.89	3.75	0.51	3.24	0.463	0.042	0.004	0.008	0.023	0.026	-0.33	1.41
d [m]	2.72	0.43	0.25	0.18	0.213	-0.042	-0.016	0.000	-0.046	0.016	-0.15	1.20
l [m]	4.14	0.62	0.15	0.46	0.158	0.042	0.074	-0.007	-0.013	-0.007	0.16	0.96

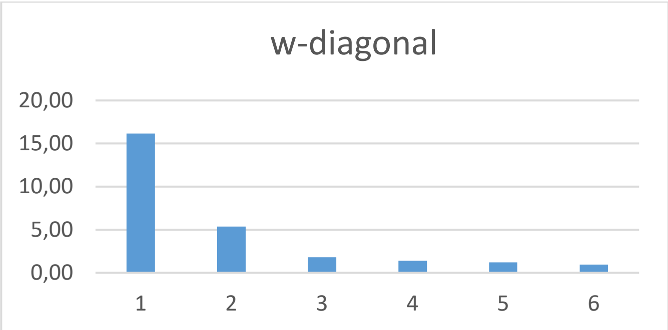


Figure 3. The influence of the SVD parameters.

It can be noted that the two first SVD parameters dominates. This can be understood in such a way that in addition to size there is also the bypass ratio that is a major design parameter that strongly influence the properties for an engine. (The sorting of the diagonal elements in the used algorithm has some issue).

Since the two first parameters are dominant a reduced model base on this is:

$$
\begin{aligned}
&\mathrm{Bpr} = 10^{0.67+0.106 s_1 - 0.208 s_2} = 4.68 \times 10^{s_1 0.106} 10^{s_2 0.208} = 4.68 \times S_1^{0.106} S_2^{0.208} \\
&T = 87.1 \times 10^{s_1 0.483} 10^{s_2 0.025} = 87.1 S_1^{0.483} S_2^{0.025} \\
&\mathrm{Sfc} = 0.457 S_1^{-0.074} S_2^{0.101} \\
&m = 1737 S_1^{0.463} S_2^{0.042} \\
&d = 1.77 S_1^{0.213} S_2^{-0.042} \\
&l = 1.41 S_1^{0.158} S_2^{0.042}
\end{aligned}
\tag{13}
$$

Here we have also introduced $S_i = 10^{s_i}$ Varying s between -1 and 1 means that S would vary between 0.1 and 10, to stay within the standard deviation of the dataset. Note that setting both S to one means that the average engine is obtained.

MODELLING OF MILITARY AIRCRAFT. ESTIMATION WITH LIMITED DATA.

Using open data, the following table of characteristics of military aircraft has been established. These data are not as exact as would be desirable but can still be used to build a statistical model of military aircraft. Since the difference in maxTOW and empty weight can be distributed differently between payload and fuel, the quotient Range/max (internal) fuel has been used instead.

Table 4. Data of some military aircraft.

Name	Service ceiling [m]	Max speed (Mach)	empty [kg]	maxTOW [kg]	Range/maxFuel	Max thrust [kN]	wingarea [m^2]	span [m]	length [m]	Stealth (1=no, 2=yes)
Typhoon	19000	2.00	10000	21000	0.925	180.0	51.2	10.5	15.96	1
Rafale C	19810	2.00	9060	15060	0.512911	174.0	46	10.9	15.3	1
PAK FA	20000	2.30	18000	35000	0.339806	334.0	78.8	13.95	19.8	2
BAE Hawk 200	15250	0.84	4128	9101	0.655882	26.0	16.69	9.39	11.38	1
F-5E	15800	1.45	4349	11214	0.438549	44.4	17.28	8.13	14.5	1
L-159	13200	0.76	4350	8000	1.01225	28.2	18.8	9.54	12.72	1
M-346	14716	0.86	4610	9500	0.9905	28.0	23.52	9.72	11.49	1
Mitsubishi F-2A	18000	2.00	9527	22100	0.137139	131.0	34.84	11.13	15.52	1
KAI T-50	14630	1.50	6470	12300	0.86093	78.7	23.69	9.45	13.4	1
Atlas Cheetah C	17000	2.20	6600	13700	0.379456	71.0	35	8.22	15.55	1
Mirage 2000	17060	2.20	7500	17000	0.338131	95.1	41	9.13	14.36	1
F-15C	19810	2.50	12975	30845	0.747148	212.0	56.48	13.05	19.43	1
Mig 29	18013	2.40	10900	20000	0.408571	184.4	38	11.36	17.32	1
Su 27S	19000	2.35	16380	23140	0.297872	245.2	62.04	14.7	21.94	1
J-10	18000	2.20	9750	19277	0.244444	130.0	39	9.75	15.49	1
JA 37	18000	2.10	9500	20000	0.5	125.0	46	10.6	16.4	1
J 35F	18000	2.20	7425	11914	0.558036	78.4	49.22	9.42	15.35	1
MIG-31	20600	2.83	21820	46200	0.459025	304.0	61.6	22.69	22.69	1
F-35A	18288	1.70	13199	31800	0.264853	191.0	42.7	10.7	15.4	2
F-22	20000	2.25	19700	38000	0.207317	312.0	78	13.56	18.9	2
JF-17	16920	1.80	6586	12500	0.866809	84.6	24.4	9.45	14.93	1
Gripen C	15240	2.00	6620	12700	0.704846	80.0	30	8.4	14.1	1
F-16C Block 50	15240	2.00	8495	19200	0.421452	127.0	27.88	9.45	15.03	1

Creating a SVD model in the same way as before, without using the F-16 data it is possible to see how well the model can predict the characteristics of the F-16 given the known wing area, span, length, engine thrust and indicating stealth or not. The values for the SVD variables are found using a solver so that the five first SVD parameters are adjusted to satisfy these five equality constraints. In addition there are constraints on the SVD variables to be within -2, and 2. This corresponds to be within two sigma of the data set, which ensures that the solution is not outside the data set. This yields an estimation that is better than 10% in all aspects. The maxTOW has the highest deviation (9%). It should be emphasized that this may as well be due to inadequate data. Also the solver was not able to satisfy all the equality constraints so that length has an error of 3%.

Table 5. SVD model of military aircraft.

	Rel error	F16 block 50	Estimate	Adjusted	Result	Average											SVD variables	w-diagonal	residual
Service ceiling [m]	0.13	15240	17190	4.24	-0.01	4.24	-0.047	0.004	-0.006	0.001	-0.004	0.001	-0.006	-0.007	0.016	0.005	0.04	2.45	0.67
Max speed (Mach)	0.07	2.00	2.14	0.33	0.07	0.26	-0.129	0.007	-0.078	0.005	0.006	0.025	0.014	0.001	-0.001	0.007	-0.20	0.94	0.34
empty [kg]	0.07	8495	7922	3.90	-0.05	3.95	-0.206	0.020	0.025	-0.014	-0.010	-0.006	0.009	-0.007	-0.012	0.002	-0.83	0.55	0.35
maxTOW [kg]	0.09	19200	17407	4.24	-0.01	4.25	-0.200	0.005	0.038	-0.025	0.011	0.036	-0.014	0.001	0.000	-0.003	-0.37	0.40	0.19
Range/maxFuel	0.07	0.42	0.45	-0.35	-0.02	-0.32	0.141	0.184	0.008	0.007	0.007	0.006	0.001	0.000	0.000	0.000	2.00	0.34	0.06
Max thrust [kN]	0.00	127	127.00	2.10	0.06	2.05	-0.325	0.039	-0.016	0.010	0.035	-0.025	-0.006	0.002	0.002	-0.001	0.00	0.25	0.06
wingarea [m^2]	0.00	27.88	27.88	1.45	-0.13	1.58	-0.181	0.030	-0.006	0.038	-0.054	0.002	-0.008	0.002	0.000	-0.002	0.00	0.17	0.06
span [m]	0.00	9.45	9.45	0.98	-0.06	1.03	-0.074	0.026	0.029	-0.047	-0.020	-0.009	0.009	0.005	0.005	0.009	0.00	0.06	0.06
length [m]	0.03	15.03	15.48	1.19	-0.01	1.20	-0.069	0.013	-0.007	-0.016	-0.007	0.000	0.019	0.000	0.007	-0.015	0.00	0.10	0.04
Stealth (1=no, 2=y	0.00	1	1.00	0.00	-0.04	0.04	-0.053	-0.021	0.067	0.049	0.015	0.008	0.016	0.001	0.003	0.003	0.00	0.09	0.00
	0.13																		

DISCUSSION AND CONCLUSIONS

It has been demonstrated that Singular Value Decomposition can be an accurate and very valuable tool for a range of application, from establishing simple models for system optimisation, modelling subsystem to estimate system characteristics based on incomplete data. This is highly useful in conceptual and in preliminary design, where complete knowledge is not yet available. It can also be useful in even earlier stages, e.g. when studying System of System scenarios, where very simple models are needed to assess usefulness of different combinations of actors in an efficient way. In this way it can also be a tool to defined requirements, since very simple models can be used to study the impact of different requirements on the design.

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