Propagation of aerodynamic sound with aeroacoustic analogies using a low dispersion numerical method

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... research where ECOlogy & ECOnomy meet

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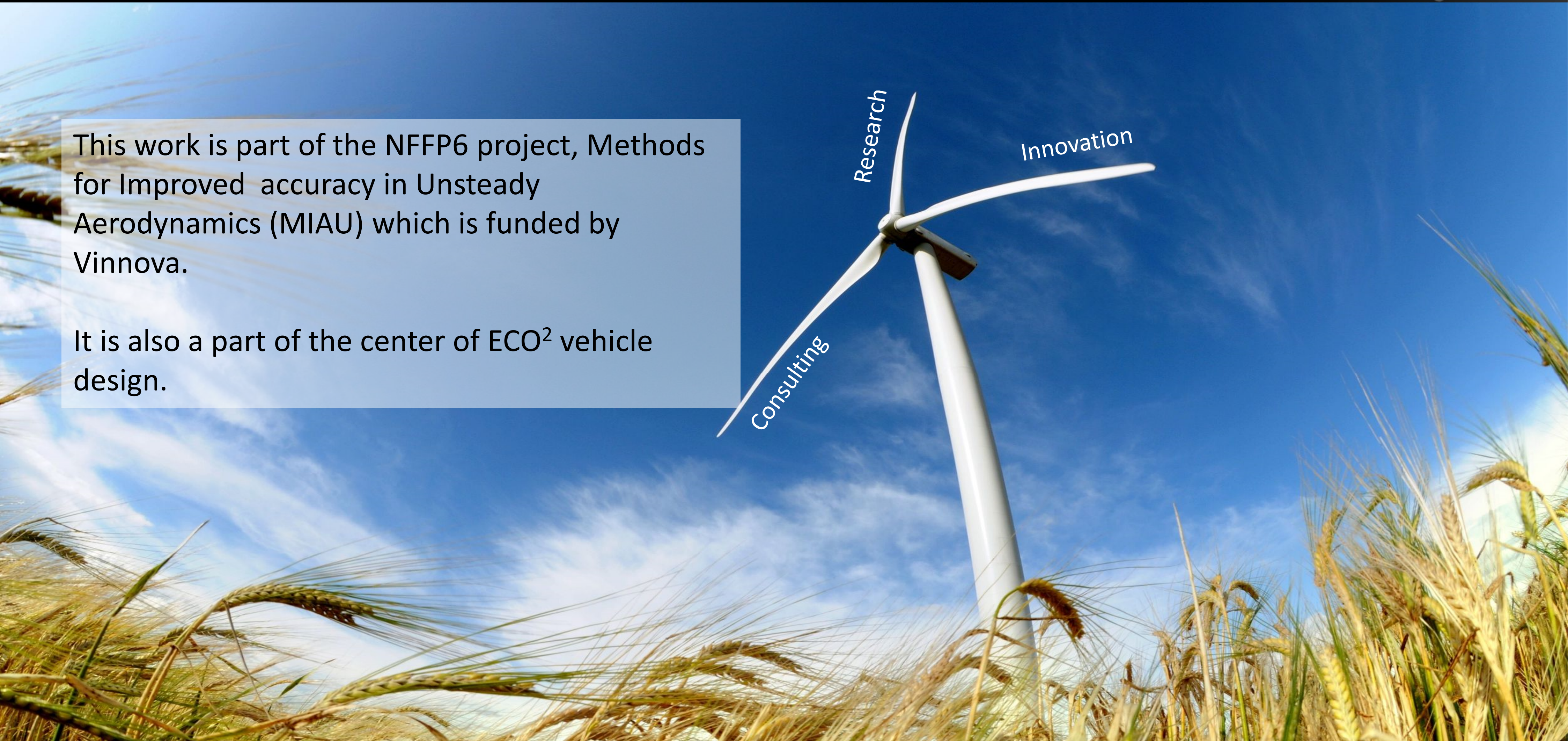
Acknowledgments

This work is part of the NFFP6 project, Methods for Improved accuracy in Unsteady Aerodynamics (MIAU) which is funded by Vinnova.

It is also a part of the center of ECO^2 vehicle design.



Overview

Objective of the presentation

- Give a short **background** to aeroacoustical problems
- Propose a method based on The **Wave Expansion Method** (WEM) that can be used to simulate the propagation of aeroacoustic sources in terms of a Boundary Value Problem (BVP)
- Give a brief **overview** of the discretization procedure
- Show the implementation of aeroacoustic **sources** in the WEM
- Solve two generic **test cases** where the procedure is used
- Give a discussion of **conclusions** regarding the procedure and results

Introduction

Computational AeroAcoustics in the Aircraft industry

- Jet noise
- Airframe noise
- Landing gear noise
- Propeller noise
- Fan noise
- Duct acoustics
- ECS noise

Introduction

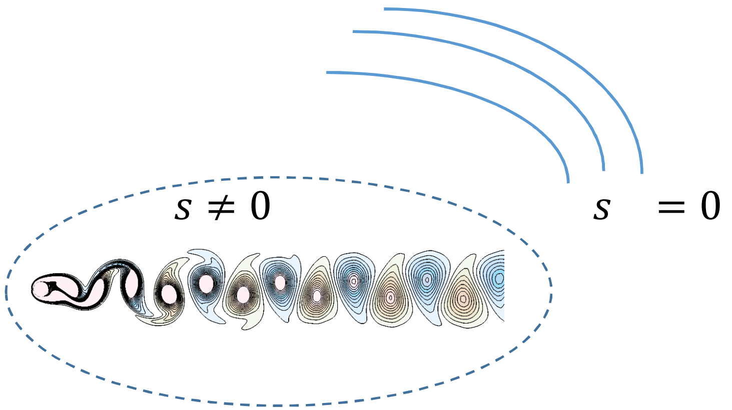
Direct simulation of flow and acoustics using the full compressible Navier-Stokes equations

- This is usually very computationally demanding
- Only possible close to the source. About 20 to 40 points/wave is needed to resolve acoustic waves in CFD
- Small acoustic perturbations tend to be dampened by CFD codes since acoustic pressures can be orders of magnitude smaller than the hydrodynamic pressure.

Use of aeroacoustic analogy.

- The concept is to rewrite the Navier-Stokes equations into an inhomogeneous wave equation which describes the source field and corresponds to a wave equation in the sound field outside the source region.

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_i} = s$$



Flow field is assumed to be uncoupled to the acoustic field!

Introduction

Flow solution, numerical(CFD) or analytical.

Compressible or incompressible flow. For almost all practical cases the flow field needs to be evaluated numerically.

Description of source, aeroacoustic analogy

Lighthill, Curle, FW-H, Vortex sound aeroacoustic analogy. Depending on the flow case considered.

Inhomogeneous wave equation solution.

Green’s function, free-field Green’s function, numerical solution of BVP.

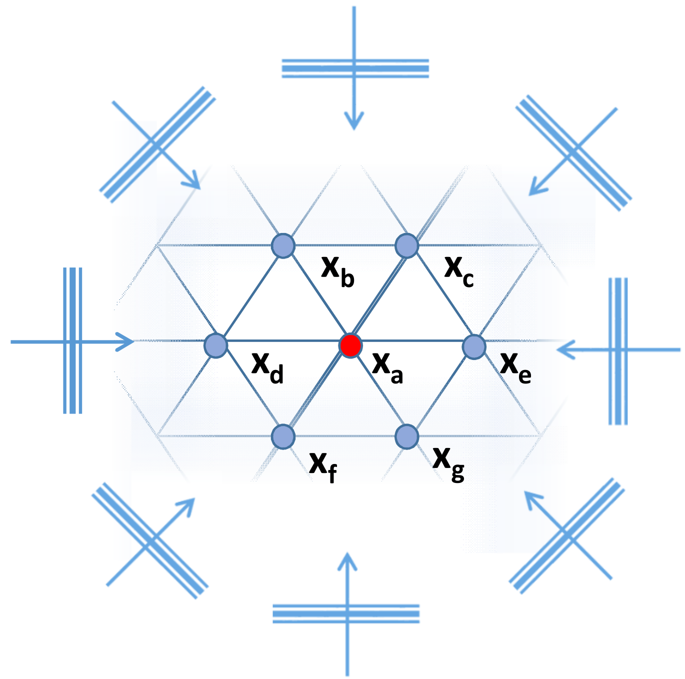
Proposed method

Aeroacoustic sources by Lighthill’s analogy and Vortex Sound Theory

Boundary value problem solved using WEM

Wave Expansion Method

- Based on a plane wave or Green's function discretization
- Discretization method for linear propagation equations
- Convective effects on acoustic waves can be included
- Not sensitive to dispersion errors
- Point based discretization
- Not restricted to certain element topologies
- Special treatment is needed for the introduction of sources



Plane waves j=1,2…J

Wave Expansion Method

The Wave Expansion Method is a discretization method suitable for solving wave equations in frequency domain. Helmholtz equation, convective Helmholtz equation, LLE.

The variable ϕ at grid point **a** can be approximated by a field of J plane waves.

$$\phi_a = \sum_{j=1}^{J} \gamma_j e^{-i\kappa \boldsymbol{\theta}_j \cdot \mathbf{x}_a} = \mathbf{h}_a \boldsymbol{\gamma}$$

$\boldsymbol{\theta}$ is the wave direction and γ the amplitude.

ϕ at neighboring grid points can be approximated by the same waves.

$$\boldsymbol{\phi}_{nb} = \mathbf{H}\boldsymbol{\gamma}$$

$$\boldsymbol{\phi}_{nb} = \left[\phi_b \;\; \phi_c \;\; \phi_d \;\; \phi_e \;\; \phi_f \;\; \phi_g \right]^T$$

$$\mathbf{H} = \left[\boldsymbol{h}_b^T \;\; \boldsymbol{h}_c^T \;\; \boldsymbol{h}_d^T \;\; \boldsymbol{h}_e^T \;\; \boldsymbol{h}_f^T \;\; \boldsymbol{h}_g^T\right]^T$$

$\boldsymbol{\gamma}$ is calculated by premultiplcating with $\mathbf{H}^+$ which is the Moore-Penrose psuedo inverse.

$$\boldsymbol{\gamma} = \mathbf{H}^+ \boldsymbol{\phi}_{nb}$$

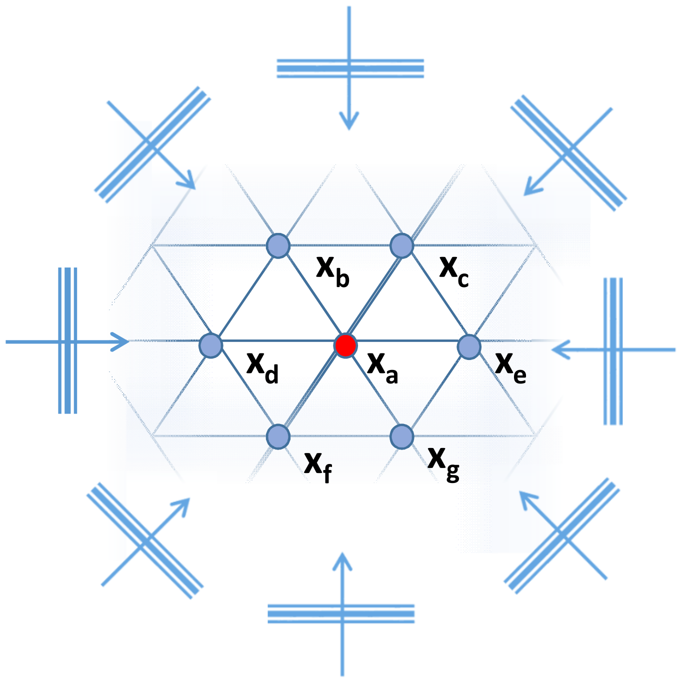
ϕ at **a** related to ***nb***

$$\phi_a = \mathbf{h}_a \mathbf{H}^+ \boldsymbol{\phi}_{nb}$$

The system is then assembled and solved

$$\mathbf{K}\,\boldsymbol{\phi} = \mathbf{Q}$$

$\mathbf{K}$ Is a global matrix of $\mathbf{h}_a \mathbf{H}^+$



Plane waves j=1,2…J

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Wave Expansion Method

Point source

- Point sources need special treatment to get the amplitude corresponding to the monopole source in the WEM.
- The source can be distributed over the near nodes by Green's function.

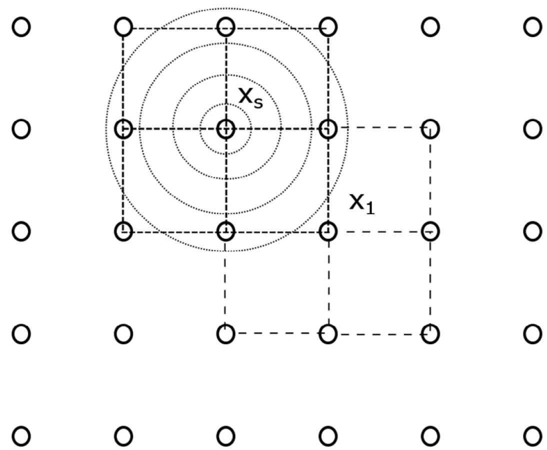
$$\phi_1 = \mathbf{h}_1 \boldsymbol{\gamma} + q$$

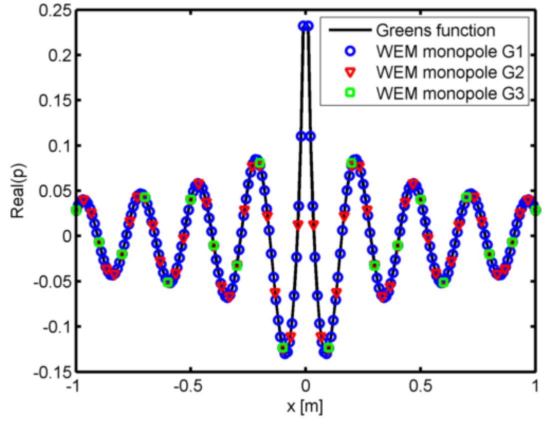
$$q = Q_s \boldsymbol{G}(\mathrm{x}_1 | \mathrm{x}_s)$$

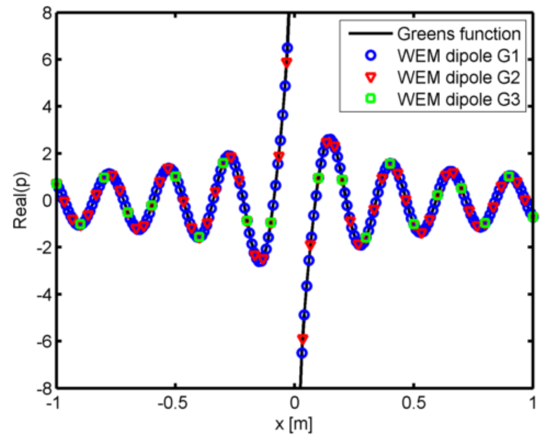
$$\boldsymbol{\phi}_n = \mathbf{h}_n \boldsymbol{\gamma} + \mathbf{Q_1}$$

$$\mathbf{Q_1} = Q_s \boldsymbol{G}(\mathrm{x}_1 | \mathrm{x}_s)$$

$$\phi_1 - \mathbf{h}_\mathrm{n} \mathrm{H}_\mathrm{n}^{+} \boldsymbol{\phi}_\mathbf{n} = q - \mathbf{h}_\mathrm{n} \mathrm{H}_\mathrm{n}^{+} \mathbf{Q_1}$$







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Source transfer

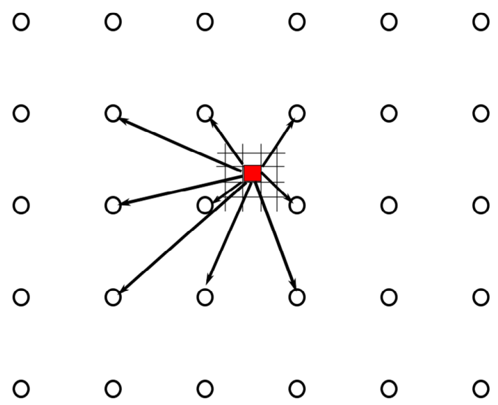
- The flow data has to be transferred to the acoustic grid which is usually coarser than the flow computation grid. The volume source of each CFD cell is therefore transferred to the appropriate nodes of the acoustic grid.

$$\mathcal{L}(\hat{p}) = \sum \hat{s}$$

- $\mathcal{L}$ is the wave operator and $\hat{s}$ is the source for the CFD node.

$$\hat{s} = \int_{\Omega} \hat{q}\, d\Omega \qquad \hat{q} = \frac{\partial^2 \hat{T}_{ij}}{\partial x_i \partial x_j} \;\; or \quad \hat{q} = \nabla \cdot \rho_0 \hat{\boldsymbol{L}}$$

- q is the distributed source from the flow computation.

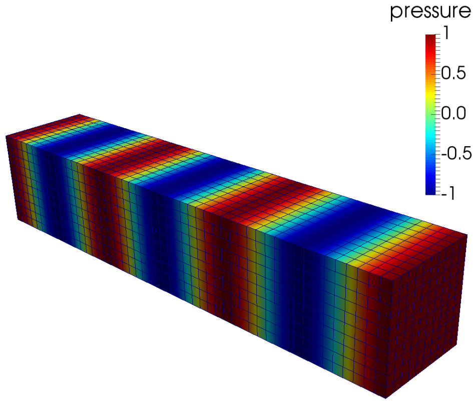


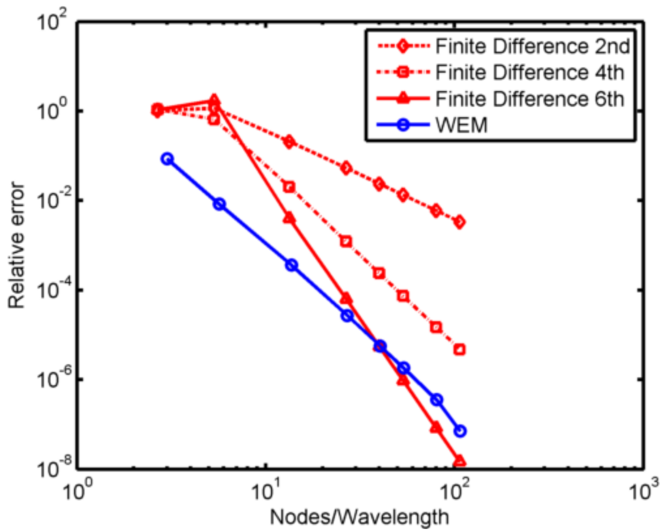
The flow sources are distributed using the appropriate free-field Green's function as required for these sources in the WEM.

Efficiency

Short remark regarding the spatial discretization efficiency

- The low dispersion error will give accurate results at just a few points per wavelength
- In terms of order the scheme is compatible with 4th order finite difference.





Co-rotating vortex pair

Flow field

- The flow is assumed to be **Incompressible**. This means that the **Lamb-Orseen vortex** velocity description of a single vortex can be used.

$$u_\theta = \frac{\Gamma}{2\pi r_y}\left(1 - e^{-r_y^2/2\sigma^2}\right)$$

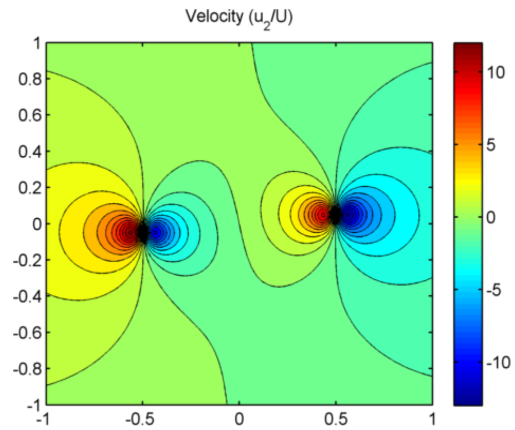
$$u_1 = -u_\theta \frac{y_2 - y_2^0}{r_y};\ u_2 = u_\theta \frac{y_1 - y_1^0}{r_y}$$

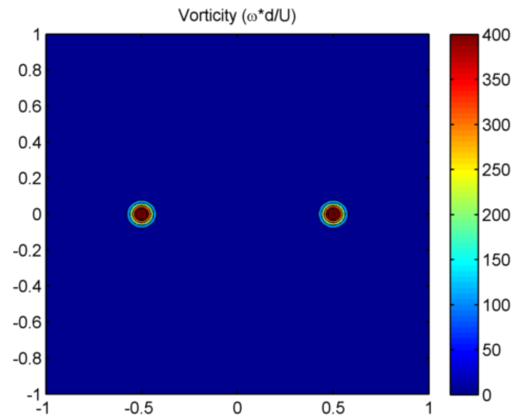
- Vortex vorticity field is described by,

$$\omega = \frac{\Gamma}{2\pi\sigma^2} e^{-r_y^2/2\sigma^2}.$$

- Rotation imposed form one vortex to the other results in a co-rotation given by,

$\Omega = 2U/d = \Gamma/(\pi d^2)$ and $M = U/a$





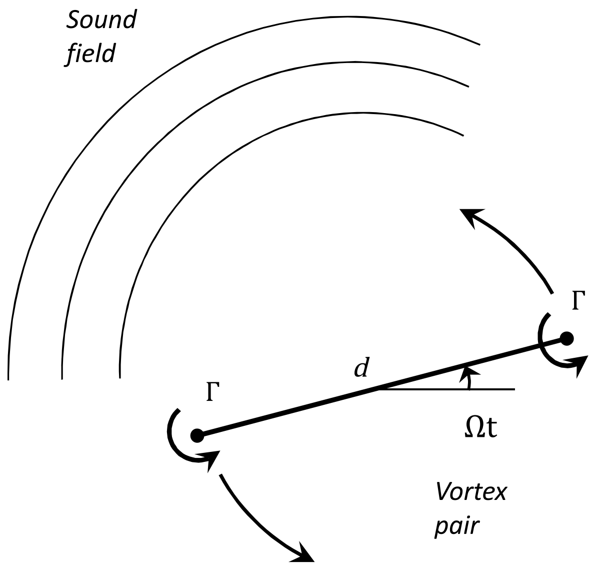
Co-rotating vortex pair

Acoustic field

- Using the vortex sound theory, Howe show that the acoustic field by two co-rotating line vortices can be described by,

$$p'(x) \approx -4\sqrt{\frac{\pi d}{2r}}\rho_0 U^2 M^{\frac{3}{2}} cos\left[2\varphi - 2\Omega\left(t - \frac{r}{c_0}\right) + \frac{\pi}{4}\right],$$

- Where M is the rotation Mach number.
- This is valid for incompressible flow and that the pressure is evaluated at a far-field location.



M.S. Howe: Theory of Vortex Sound,
Cambridge Texts in Applied Mathematics, 2003.

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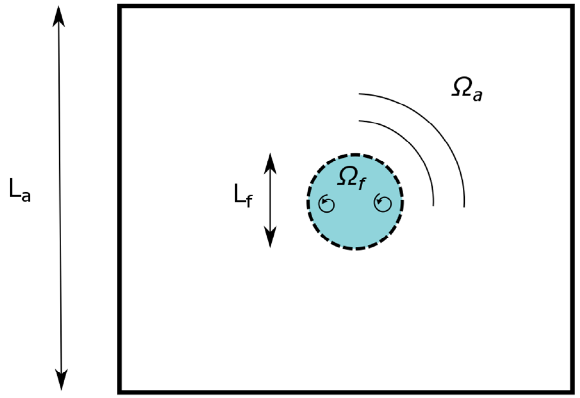
Co-rotating vortex pair

Inhomogeneous wave equation

- For a receiver location in a stagnant flow,

$$\nabla^2 \hat{p} + k^2 \hat{p} = \hat{s}$$

- Where k is the wave number and $\hat{p}$ is the acoustic pressure.
- This can be solved as a boundary value problem imposing the appropriate boundary conditions.



Source description

- Based on the analogy of Lighthill the following formulation of $\hat{s}$ is given.

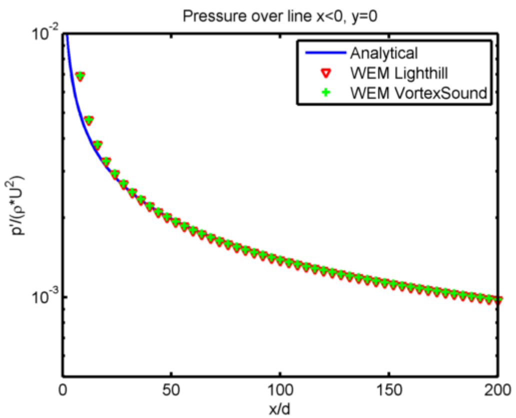
$$\hat{s} = \frac{\partial^2 \hat{T}_{ij}}{\partial x_i \partial x_j} \qquad T_{ij} = \rho_0 u_i u_j$$

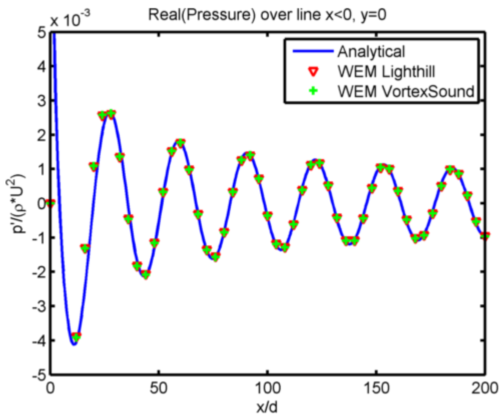
- For low Mach number flow where entropy effects are neglected.
- Based on the theory of vortex sound the source is formulated based on the vorticity.

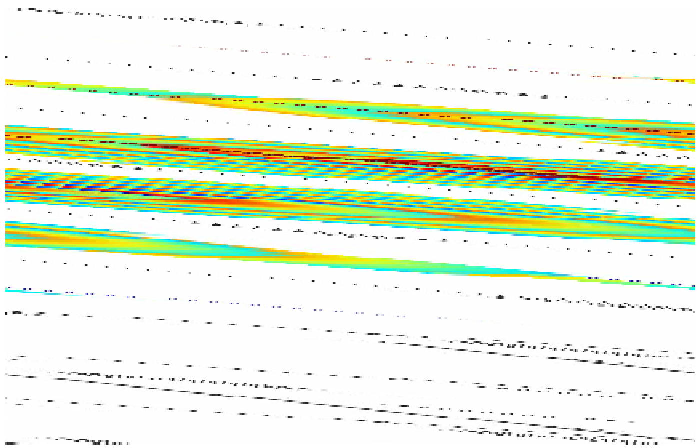
$$\hat{s} = \nabla \cdot \rho_0 \hat{\boldsymbol{L}} \qquad \boldsymbol{L} = (\boldsymbol{\omega} \times \boldsymbol{u})$$

Results of acoustic propagation

- Pressure presented on a line from the center of rotation
- The two analogies gives more or less exactly the same result although the formulation of the sources are not identical.
- Good agreement with the analytical solution.



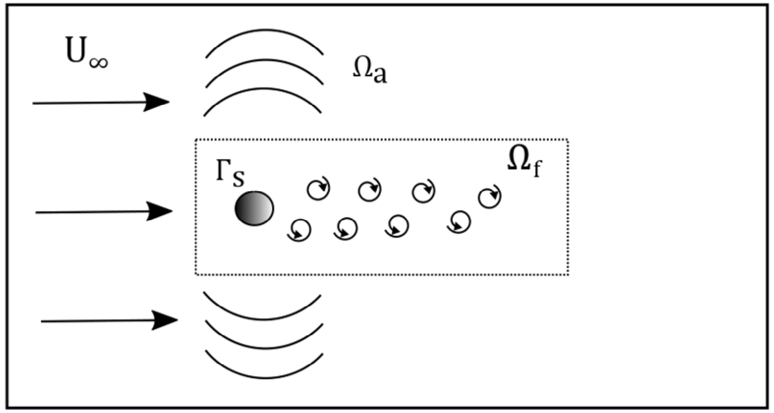


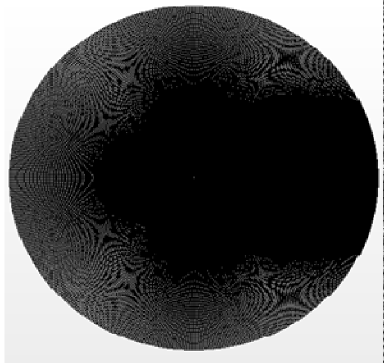


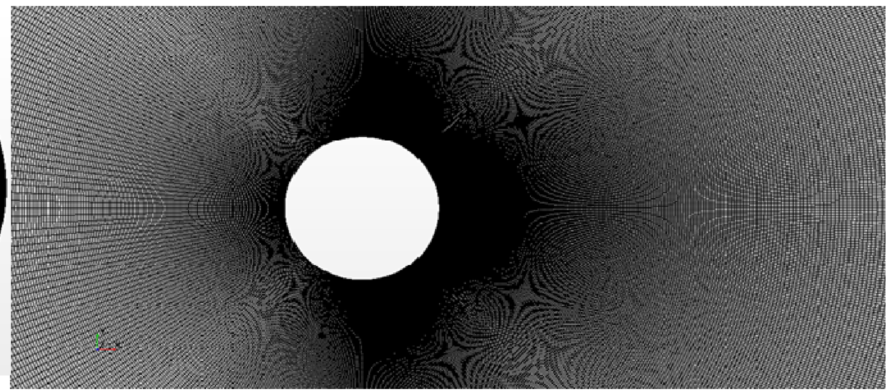
Cylinder in cross-flow

Flow field

- Star CCM+ v10
- Mach number 0.1, 0.2 and 0.3
- Reynolds number 150
- In these flow conditions the flow field can be computed as laminar.
- Three different grid levels considered
- Finest grid used in the evaluation
- Goal of this case is to evaluate a more extended source region with a background flow





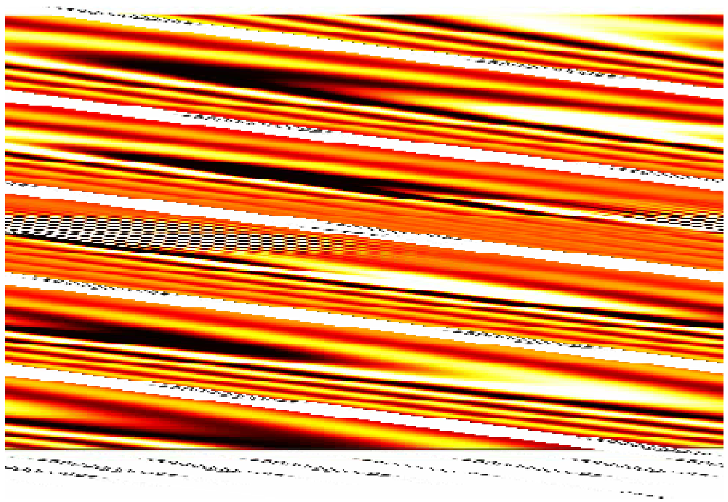


Cylinder in cross-flow

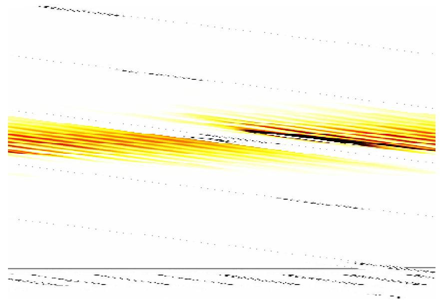
Direct computation of flow and acoustic field

- Pressure fluctuations of CFD computations
- Presented on the fine grid
- Mach = 0.3
- Lighthill sources given on the right are used with the surface pressures in the WEM.

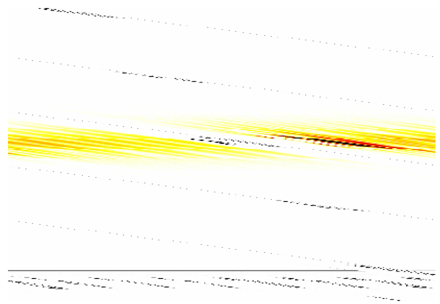
Pressure



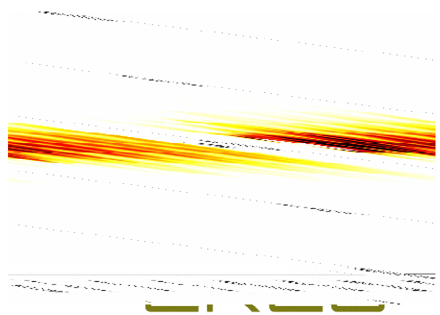
T_{11}



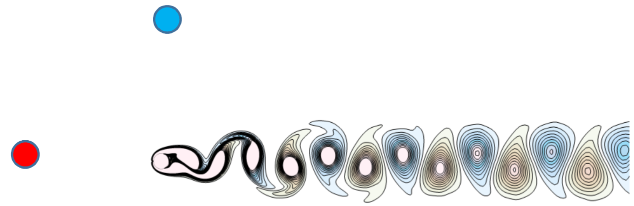
T_{12}



T_{22}

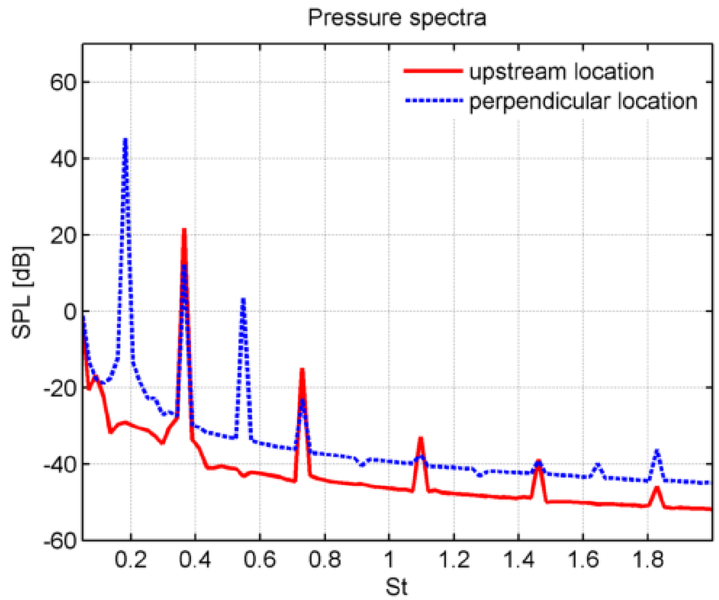


Cylinder in cross-flow



Direct computation of flow and acoustic field

- The vortex shedding gives distinct tonal components
- Shedding frequency has a strong directivity in the lift force direction.
- The first harmonic will correspond to the drag force oscillations an has a strong directivity upstream



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Cylinder in cross-flow

Acoustic field

- The FWH aero acoustic analogy can be formulated as a inhomogeneous convected Helmholtz equation

$$\left\{\frac{\partial^2}{\partial x_i^2} + k^2 - 2iM_i k\frac{\partial}{\partial x_i} - M_i M_j \frac{\partial^2}{\partial x_i \partial x_j}\right\}[H(f)c_\infty^2 \rho'(\mathbf{x},\omega)] = -\frac{\partial^2}{\partial x_i \partial x_j}\left[T_{ij}(\mathbf{y},\omega)H(f)\right] - \frac{\partial}{\partial x_i}[F_i(\mathbf{y},\omega)\delta(f)].$$

- This can be solved as an integral over all sources

$$H(f)p'(\mathbf{x},\omega) = -\iint_{f>0} T_{ij}(\mathbf{y},\omega)\frac{\partial^2 \mathrm{G}(\mathbf{x}|\mathbf{y},\omega)}{\partial y_i \partial y_j}d\mathbf{y} + \int_{f=0} F_i(\mathbf{y},\omega)\frac{\partial \mathrm{G}(\mathbf{x}|\mathbf{y},\omega)}{\partial y_i}d\Sigma(\mathbf{y})$$

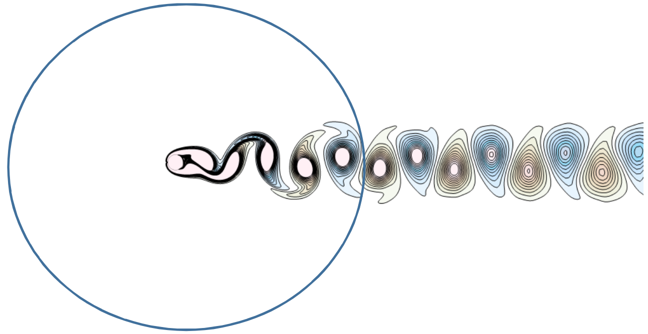
- The source terms will then be in the form

$$T_{ij} = \rho(u_i - U_i^\infty)(u_j - U_j^\infty) + (p - c_\infty^2\rho)\delta_{ij} + \tau_{ij}$$ *Lighthill stress tensor (Quadropole)*

$$F_i = (p\delta_{ij} - \tau_{ij})n_j$$ *Surface force (Dipole)*

X. Gloerfelt, C. Bailly, D. Juv´e, *Direct computation of the noise radiated by a subsonic cavity flow and application of integral methods*, Journal of Sound and Vibration 266 (2003) 119–146

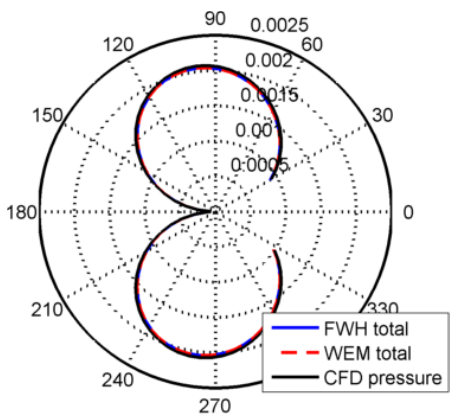
Cylinder in cross-flow



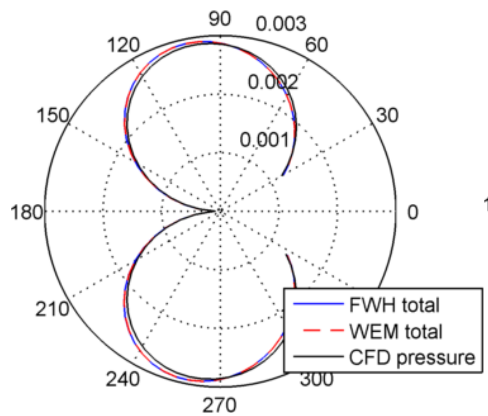
WEM solution for propagation of sources

- Amplitude of pressure evaluated on a circle surrounding the cylinder for the three different Mach numbers
- FWH integral solution is also included which gives the same results as WEM
- Some deviation in the amplitude for Mach = 0.3

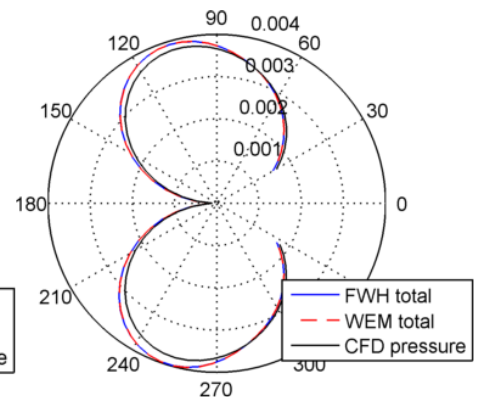
Mach = 0.1



Mach = 0.2



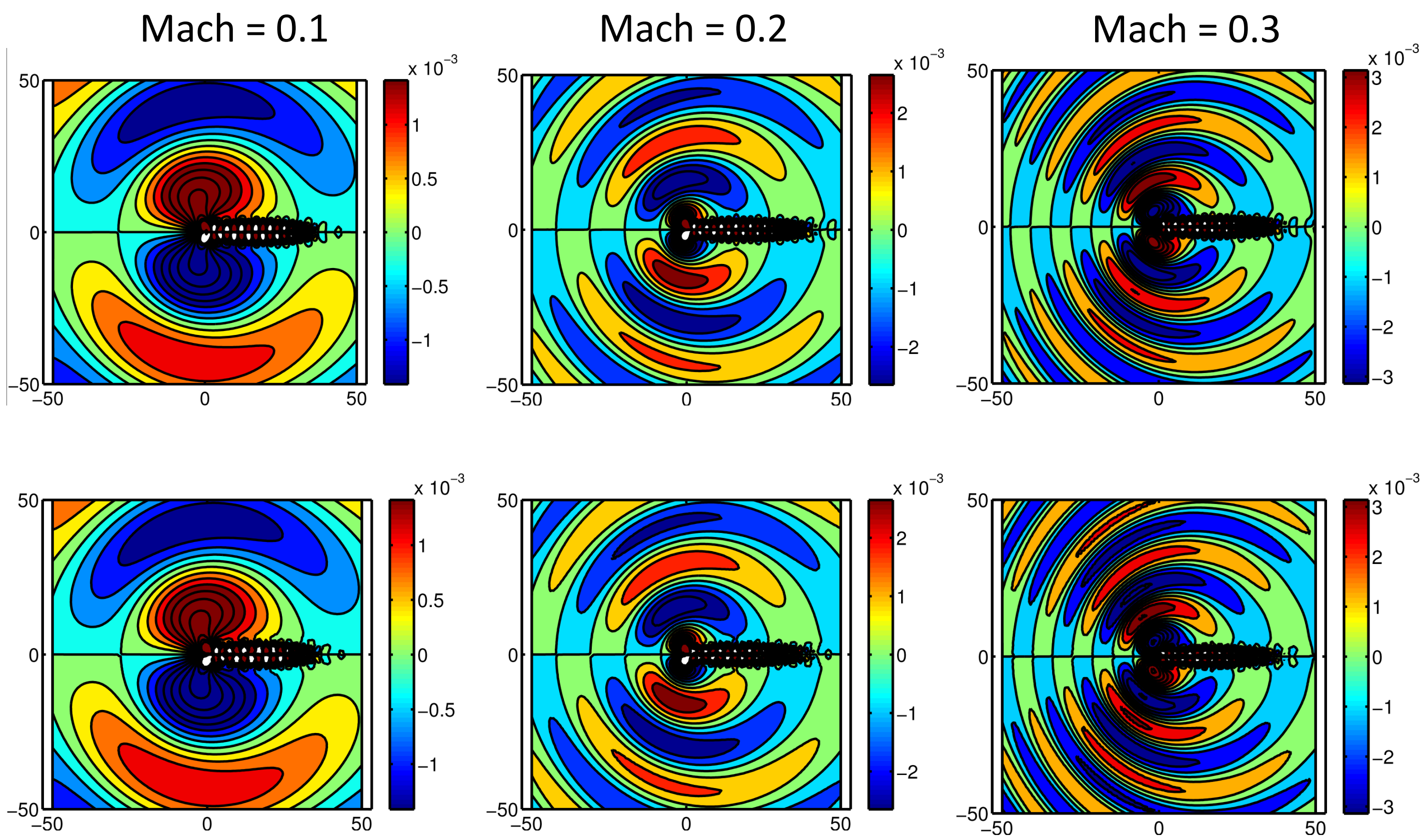
Mach = 0.3



Cylinder in cross-flow

WEM solution for propagation of sources

- Pressure evaluated on a circle surrounding the cylinder for the three different Mach numbers
- The solution based on the aeroacoustic analogy solved using WEM show results that are very close to the CFD results.
- The slight difference at Mach=0.3 could also be related to the CFD grid being to course



Conclusions

Currently

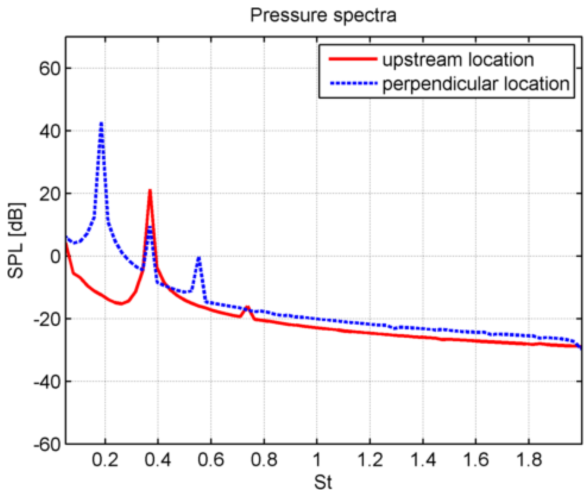
- CFD and analytical methods have been used to describe the aeroacoustic sources.
- These are introduced as a cloud of point sources in the WEM which enables it to be used for the propagation.
- For the problems considered reasonable agreement is shown between the different methods.
- Though this shows that it is possible to include sources and solve the radiation problem. The full use of the method will be when using it in situations where scattering by objects occurs or were the propagation is influenced by an inhomogeneous flow.

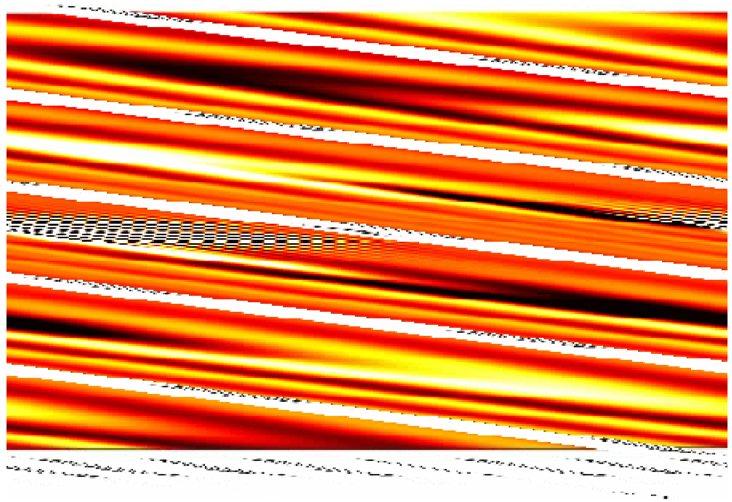
Future

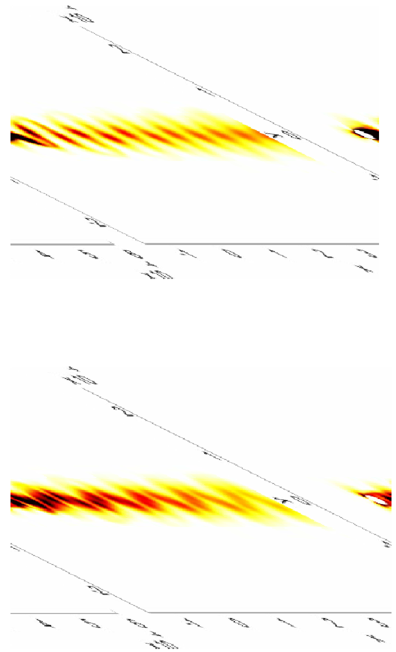
- Solve for velocity potential in a potential flow.
- Implementation of 3D which should not be any problem due to the point based discretization.
- Scattering by walls.

Thank you for the attention!

Direct computation of flow and acoustic field







Wave Expansion Method

Plane-wave solutions

$$\phi = \mathbf{v}e^{-ik(\boldsymbol{\theta}\cdot\mathbf{x})}$$

$\mathbf{v}$ *is the eigen vector*

k *is the wave number*

θ *is the direction vector*

Helmholtz

$$\left\{\boldsymbol{\nabla}^2 + \frac{\omega^2}{c^2}\right\}\hat{p} = 0$$

$$\mathbf{v_1} = 1 \qquad \mathrm{k}_1 = \frac{\omega}{c}$$

$$\mathbf{v_2} = -1 \qquad \mathrm{k}_2 = -\frac{\omega}{c}$$

Convected Helmholtz

$$\left\{\boldsymbol{\nabla}^2 - \left(\frac{1}{c}\mathbf{u}\cdot\boldsymbol{\nabla}\right)^2 - \frac{2i\omega}{c^2}\mathbf{u}\cdot\boldsymbol{\nabla} + \frac{\omega^2}{c^2}\right\}\hat{p} = 0$$

$$\mathbf{v_1} = 1 \qquad \mathrm{k}_1 = \frac{\omega}{\mathbf{u}\cdot\boldsymbol{\theta} + c}$$

$$\mathbf{v_2} = -1 \qquad \mathrm{k}_2 = \frac{\omega}{\mathbf{u}\cdot\boldsymbol{\theta} - c}$$

Coupled mass momentum

$$i\omega\hat{p} + \rho c^2\boldsymbol{\nabla}\cdot\hat{\mathbf{u}} = 0$$

$$i\omega\hat{\mathbf{u}} + \frac{1}{\rho}\boldsymbol{\nabla}\hat{p} = 0$$

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ \theta_x/(\rho c) \\ \theta_y/(\rho c) \\ \theta_z/(\rho c) \end{bmatrix} \qquad \mathrm{k}_1 = \frac{\omega}{c}$$

$$\mathbf{v_2} = \begin{bmatrix} -1 \\ \theta_x/(\rho c) \\ \theta_y/(\rho c) \\ \theta_z/(\rho c) \end{bmatrix} \qquad \mathrm{k}_2 = -\frac{\omega}{c}$$

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