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AN INVARIANT-BASED DESIGN APPROACH TO CARBON FIBER REINFORCED POLYMER COMPOSITE LAMINATES

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Outline

- Motivation
  - Invariants
  - Trace
    - Master Ply
  - Unit Circle
- Testing and Design with Trace and Unit Circle
  - Testing
  - Designing
- Conclusions

MOTIVATION

- Tsai and Pagano (1967)*:
  - Invariants to describe stiffness transformation equations for ply rotation
- Invariants are not affected by ply orientation
  - Material property
  - Possibility for testing laminates (instead of ply)
    - Reduced number of tests: cheaper and faster
  - Improve optimization

*S.W. Tsai, N.J. Pagano. “Invariant Properties of Composite Materials”. *In: Composite Materials Workshop*, S.W. Tsai, J.C. Halpin and N.J. Pagano (Editors), St. Louis, Missouri, 1967, Technomic Publishing Company, 1968, p. 233-253.

Invariant - Trace

$$\mathrm{Tr}\,[Q] = Q_{xx} + Q_{yy} + 2Q_{ss} = Q_{11} + Q_{22} + 2Q_{66}$$

where: Q_{xx}, Q_{yy}, Q_{ss} and Q_{11}, Q_{22}, Q_{66} are the on-axis and off-axis plane stress stiffness components, respectively

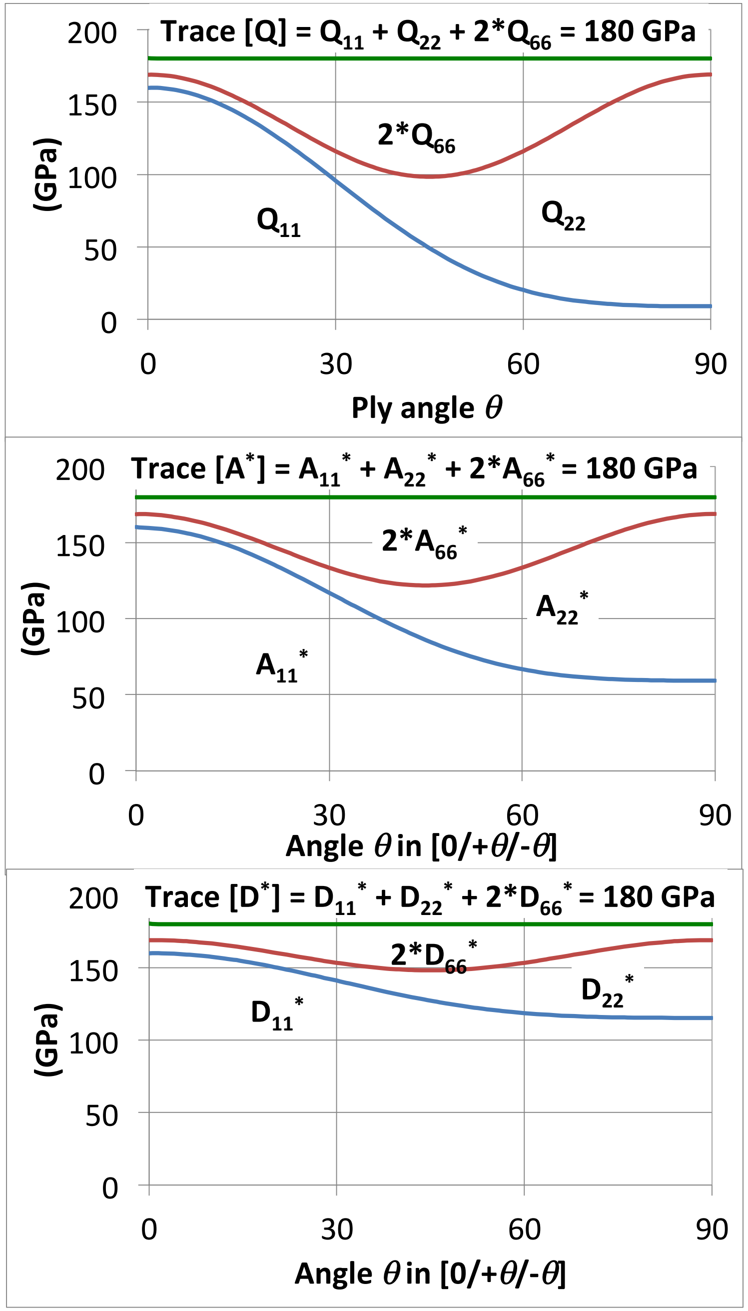
$$\mathrm{Tr}\,[A^*] = \mathrm{A}^*_{11} + \mathrm{A}^*_{22} + 2\mathrm{A}^*_{66} \qquad [A^*] = \frac{1}{h}[\mathrm{A}]$$

$$\mathrm{Tr}\,[D^*] = \mathrm{D}^*_{11} + \mathrm{D}^*_{22} + 2\mathrm{D}^*_{66} \qquad [D^*] = \frac{12}{h^3}[\mathrm{D}]$$

- components of [A*] <u>are not</u> dependent on stacking sequence
- components of [D*] <u>are</u> dependent on stacking sequence
  - both have the same trace (invariant to stacking sequence)
- Trace is independent of the loading condition (in-plane versus flexural) and stacking sequence (mid-plane symmetric versus asymmetric)

TRACE

Tr [Q] = Tr [A*] = Tr [D*]



Unidirectional ply and in-plane and flexural stiffness components as a function of ply orientation for IM7/8552

MASTER PLY

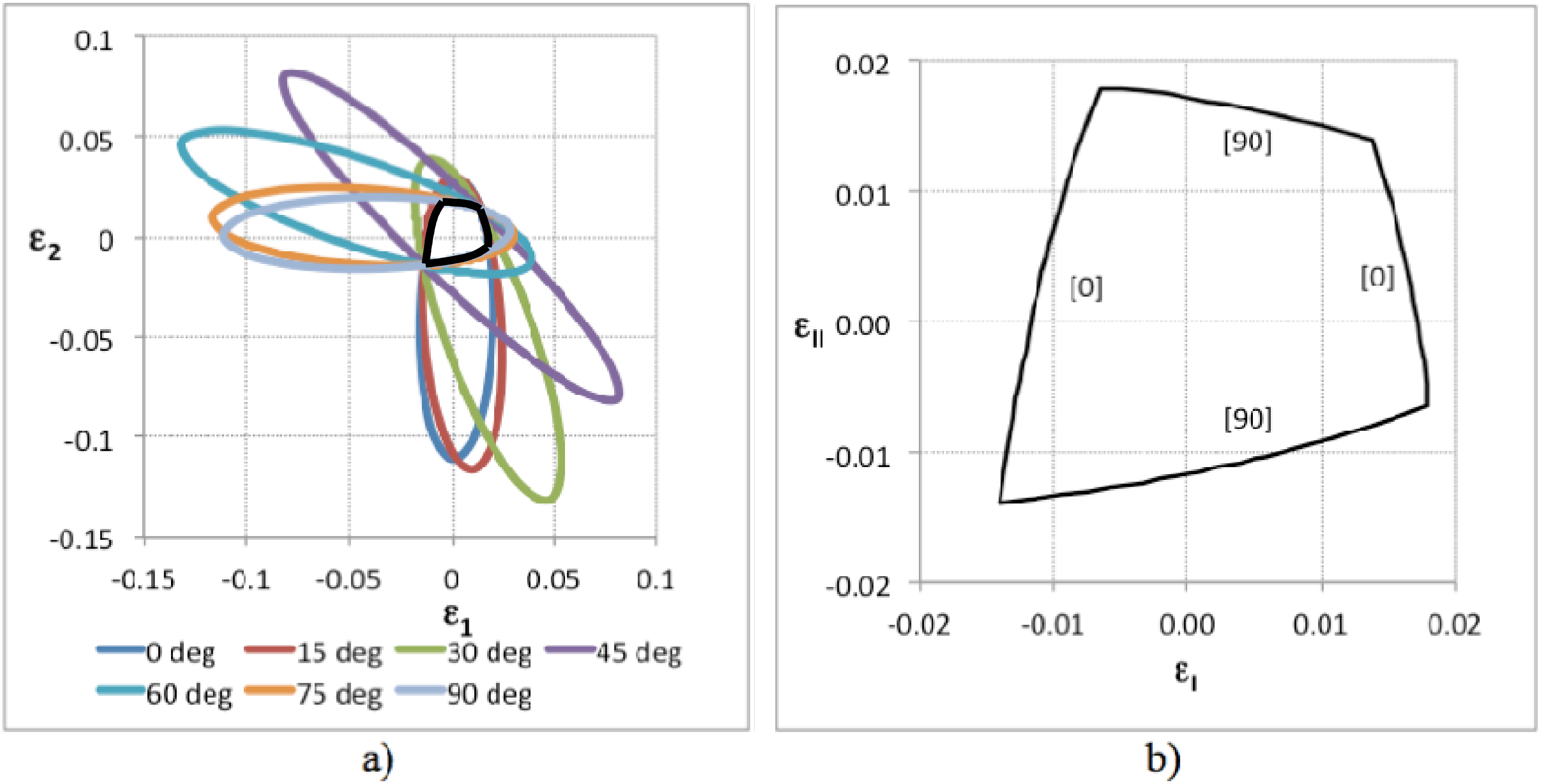
Trace normalized plane stress stiffness components for various CFRP

Material	E_x (GPa)	E_y (GPa)	ν_x	E_s (GPa)	Q_{xx}^*	Q_{yy}^*	Q_{xy}^*	Q_{ss}^*	Tr (GPa)
IM6/epoxy	203	11.20	0.32	8.40	0.8791	0.0485	0.0155	0.0362	232
IM7/977-3	191	9.94	0.35	7.79	0.8825	0.0459	0.0161	0.0358	218
T300/5208	181	10.30	0.28	7.17	0.8805	0.0501	0.0140	0.0347	206
IM7/MTM45	175	8.20	0.33	5.50	0.9014	0.0422	0.0139	0.0282	195
T800/Cytec	162	9.00	0.40	5.00	0.8955	0.0497	0.0199	0.0274	183
IM7/8552	159	8.96	0.32	5.50	0.8888	0.0501	0.0160	0.0306	180
T800S/3900	151	8.20	0.33	4.00	0.9034	0.0491	0.0162	0.0238	168
T300/F934	148	9.65	0.30	4.55	0.8878	0.0579	0.0174	0.0271	168
T700 C-Ply 64	141	9.30	0.30	5.80	0.8713	0.0575	0.0172	0.0356	163
AS4/H3501	138	8.96	0.30	7.10	0.8567	0.0556	0.0167	0.0438	162
T650/epoxy	139	9.40	0.32	5.50	0.8724	0.0590	0.0189	0.0343	160
T4708/MR60H	142	7.72	0.34	3.80	0.9029	0.0491	0.0167	0.0240	158
T700/2510	126	8.40	0.31	4.20	0.8827	0.0588	0.0182	0.0292	144
AS4/MTM45	128	7.93	0.30	3.65	0.8939	0.0554	0.0166	0.0253	144
T700 C-Ply 55	121	8.00	0.30	4.70	0.8746	0.0578	0.0173	0.0338	139
Std dev	24.6	1.0	0.029	1.5	0.0132	0.0053	0.0016	0.0056	
Coeff var %	16.0	10.9	9.0	27.2	1.5	10.1	9.6	17.9	
Master ply					0.8849	0.0525	0.0167	0.0313	1.0

Thus, CFRPs have universal trace normalized stiffness constants
Trace is the only material property needed to define ply stiffness

UNIT CIRCLE

Last-ply failure envelopes

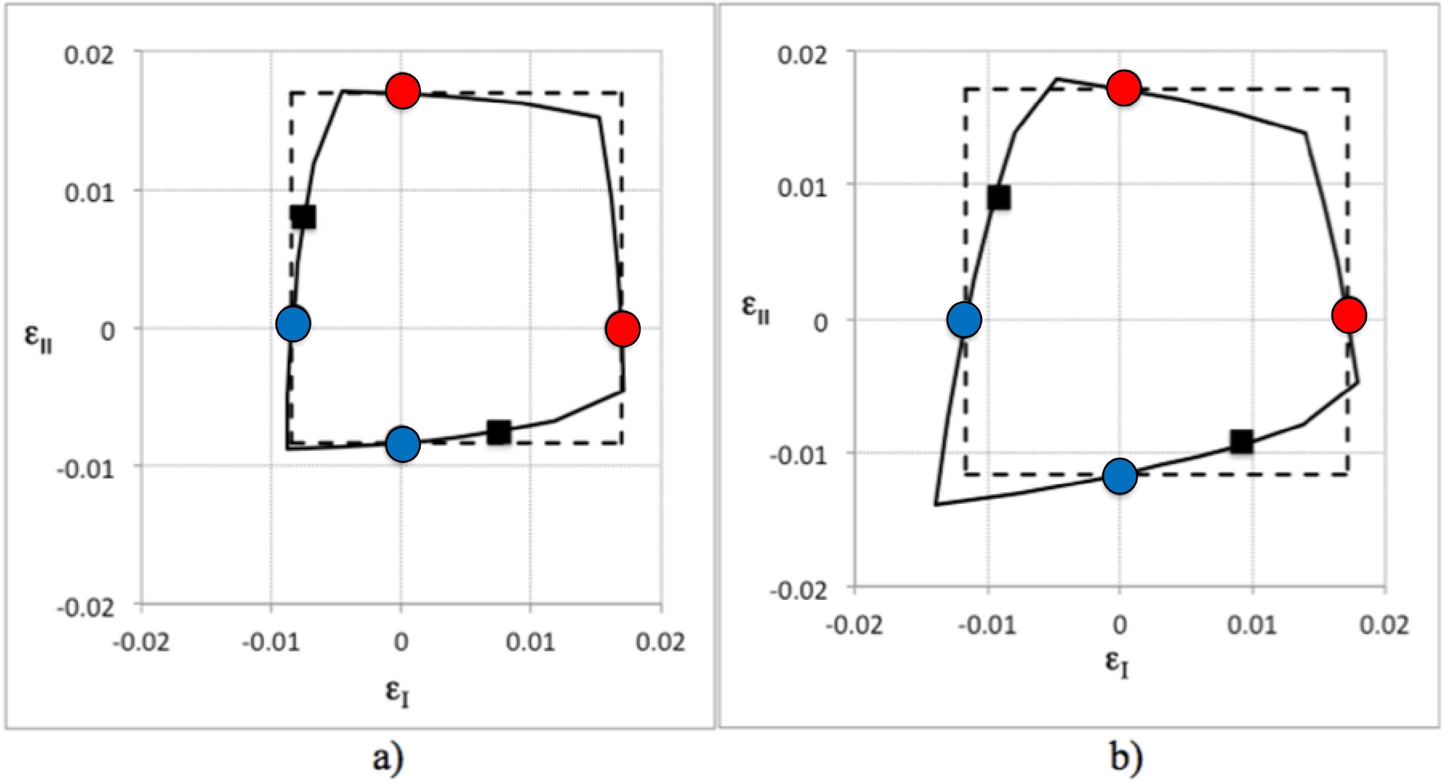


a) Tsai-Wu LPF envelopes for T700/2510 b) Omni strain LPF envelope for T700/2510

Ref. Tsai SW and Melo JDD. A unit circle failure criterion for carbon fiber reinforced polymer composites. *Composites Science and Technology* 123 (2016) 71-78.

UNIT CIRCLE

Omni strain LPF envelopes for two CFRPs based on Tsai-Wu (solid line) and maximum strain (dashed line)

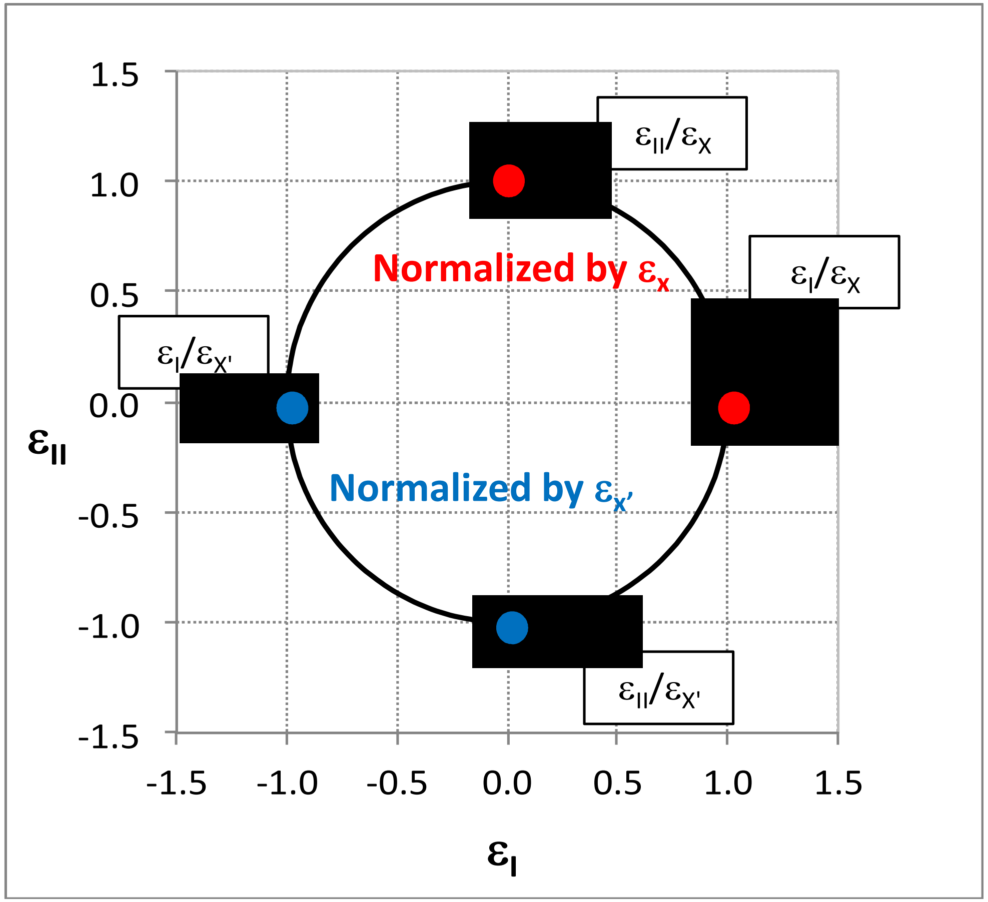


a) IM7/977-3 b) T700/2510

Same Anchor Points

Ref. Tsai SW and Melo JDD. A unit circle failure criterion for carbon fiber reinforced polymer composites. *Composites Science and Technology* 123 (2016) 71-78.

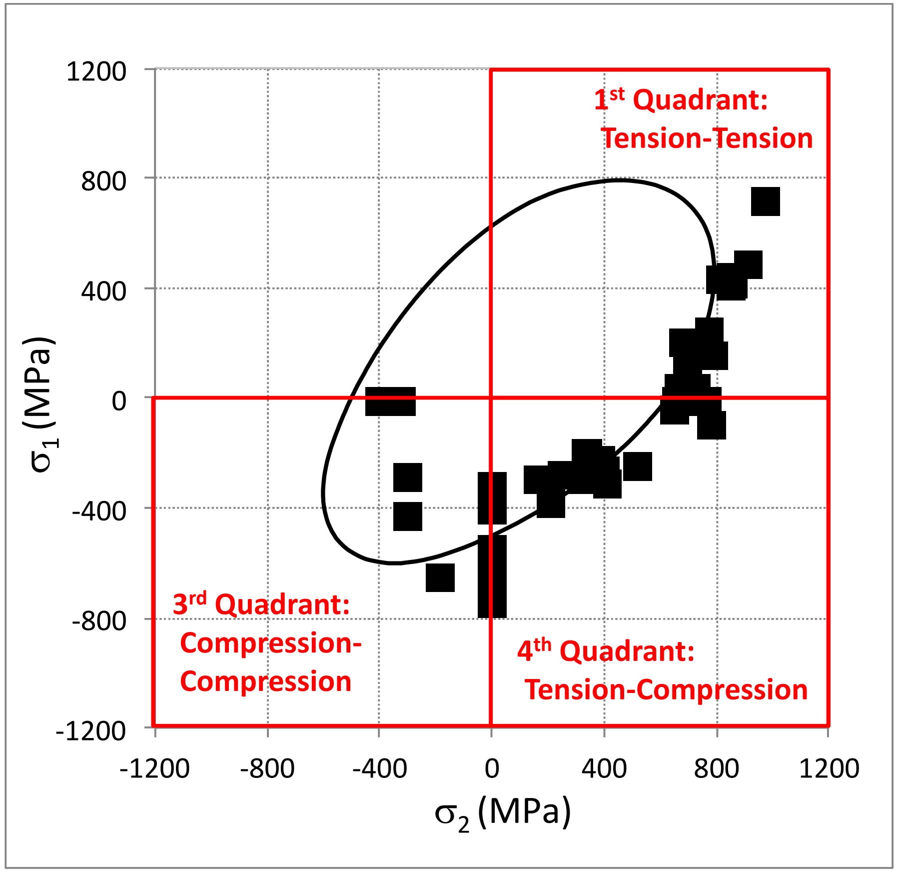
UNIT CIRCLE



Ref. Tsai SW and Melo JDD. A unit circle failure criterion for carbon fiber reinforced polymer composites. *Composites Science and Technology* 123 (2016) 71-78.

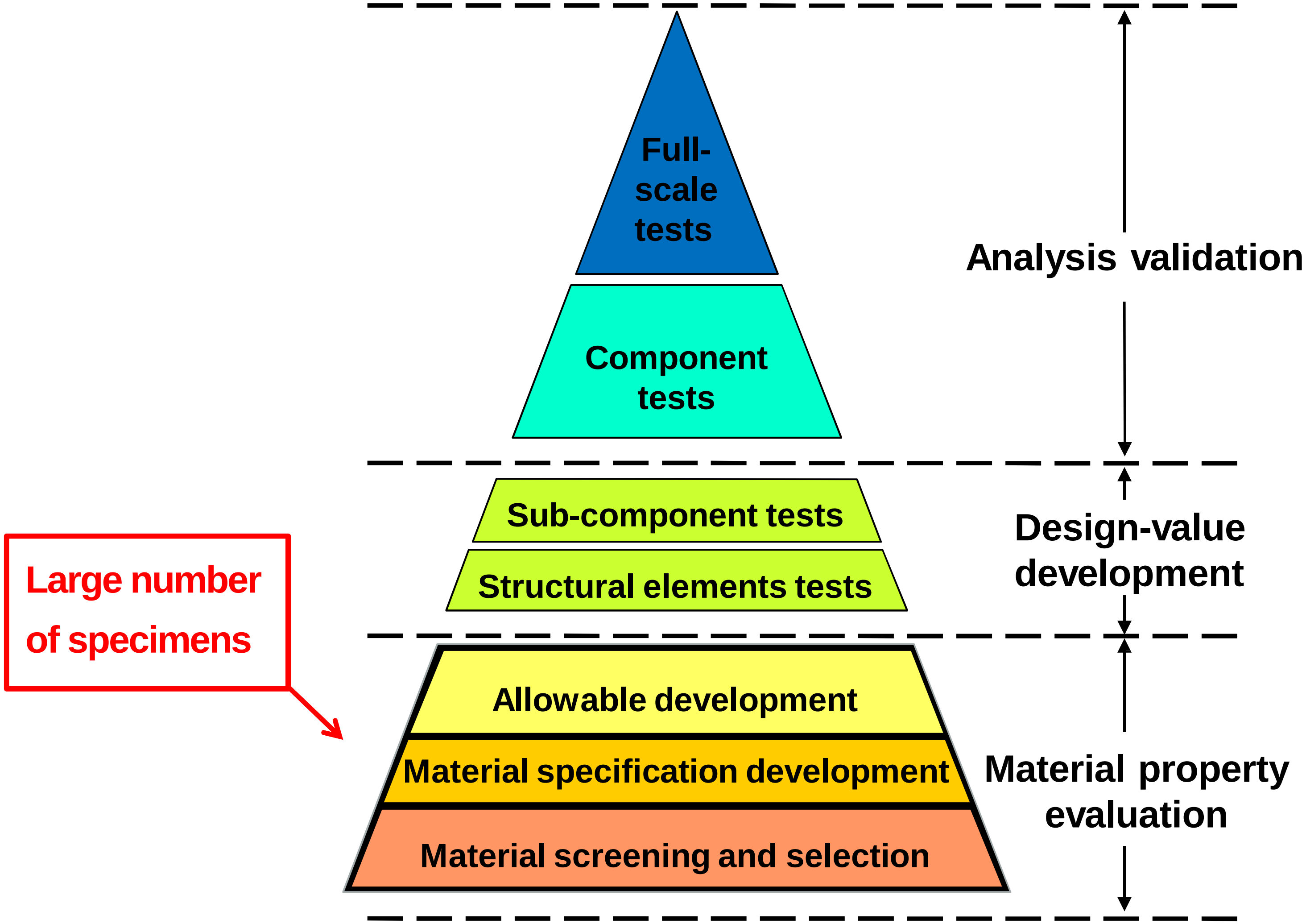
UNIT CIRCLE

Biaxial failure stress for [0/±45/90]s AS4/3501-6 laminate
Experimental data (WWFE) and unit circle failure envelope



Ref. Tsai SW and Melo JDD. A unit circle failure criterion for carbon fiber reinforced polymer composites. *Composites Science and Technology* 123 (2016) 71-78.

TESTING WITH TRACE & UNIT CIRCLE



Ref.: S.E. Cregger, Breaking the development cost curve by using analysis, *JEC Europe*, Paris, 2015

MASTER PLY

DESIGN AND TESTING

Elastic constants for master ply:

Material	E_x^*	E_y^*	ν_x	E_s^*
Master Ply	0.8796	0.0522	0.3181	0.0313

Testing with Trace

UD Coupons

$$Tr = \frac{E_X}{0.8796}$$

Laminates

$$Tr = \frac{E_1^o\ (laminate)}{E_1^o(trace\ of\ normalized\ master\ ply)}$$

- Once trace is determined, the other elastic parameters are determined based on master ply properties
- The [0] UD coupon is easy to make and quality is usually high because there are no off-axis plies
- Laminates will produce “as-built” properties

TESTING FOR UNIT CIRCLE

UD Coupons

Determine strains-to-failure under tension and compression (only two types of test)

- Same specimen used for the determination of trace

DESIGNING W/ TRACE & UNIT CIRCLE

[redacted]

[redacted]

Ref. Melo JDD, Bi J and Tsai SW. A novel invariant-based design approach to carbon fiber reinforced laminates. *Composite Structures* 159 (2017) 44–52.

TRACE

Rows: for given material, all laminates have the same trace:

144 GPa for AS4/MTM45 or 218 GPa for IM7/977-3

Columns: same universal [A*] for the same laminate

Laminate / Material	Quasi-iso: $[\pi/4]$	Hard: $[0_5/\pm 45_2/90]$
AS4/ MTM45	$144 \begin{bmatrix} 0.37 & 0.12 & 0 \\ & 0.37 & 0 \\ & & 0.13 \end{bmatrix}$ A_{11}: 0.37 A_{22}: 0.37 A_{66}: 0.13 + 0.13 Trace: 1.00	0.56 0.22 0.11 0.11 1.00 $144 \begin{bmatrix} 0.56 & 0.10 & 0 \\ & 0.22 & 0 \\ & & 0.11 \end{bmatrix}$
IM7/ 977-3	$218 \begin{bmatrix} 0.37 & 0.12 & 0 \\ & 0.37 & 0 \\ & & 0.13 \end{bmatrix}$ A_{11}: 0.37 A_{22}: 0.37 A_{66}: 0.13 + 0.13 Trace: 1.00	0.56 0.22 0.11 0.11 1.00 $218 \begin{bmatrix} 0.56 & 0.10 & 0 \\ & 0.22 & 0 \\ & & 0.11 \end{bmatrix}$

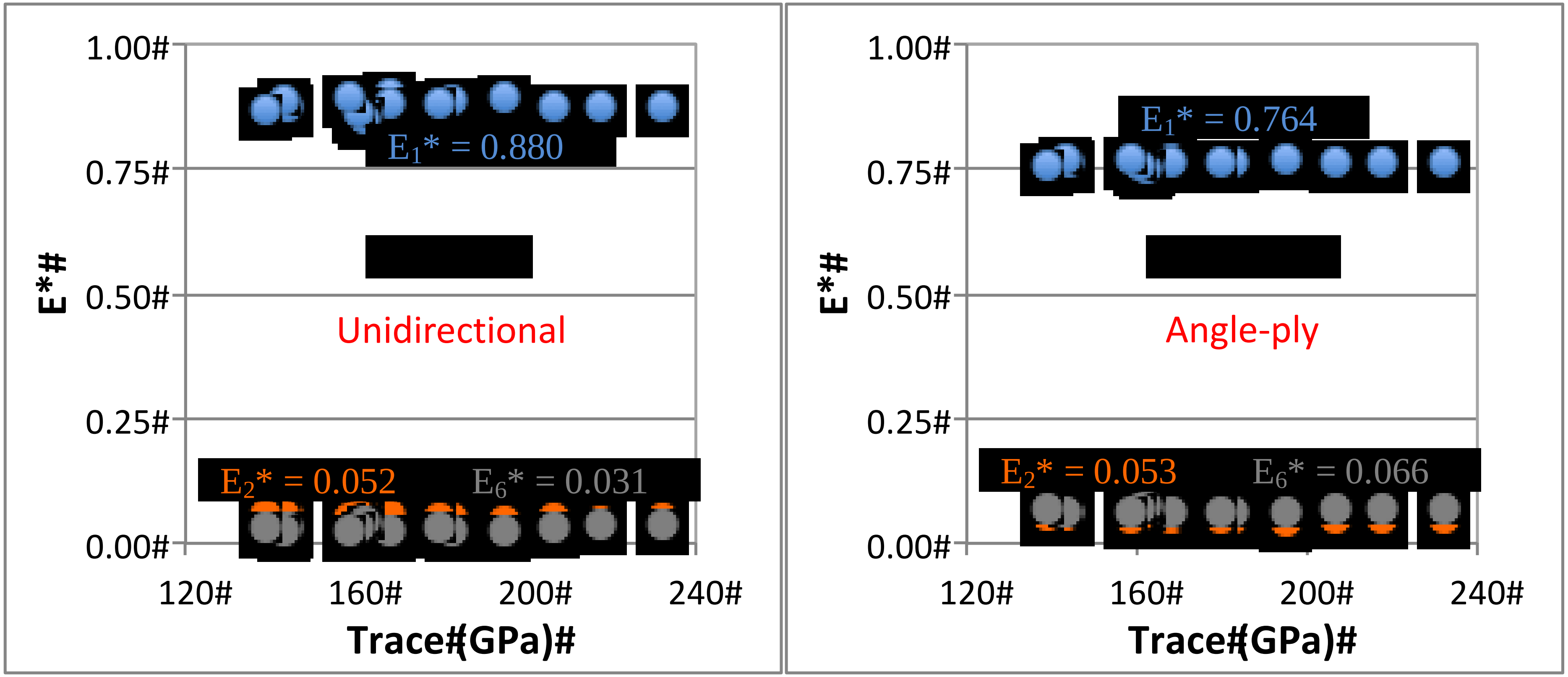
DESIGNING WITH TRACE

Longitudinal stiffness of unidirectional ply and five laminates for 15 carbon/polymer materials

Laminate	[0] UD	[±12.5] Angle-ply	$[0_2/\pm 25]$ Angle-ply (hard)	$[0_2/\pm 45]$ Angle-ply (hard)	$[0_5/\pm 45_2/90]$ QI (hard)	[0/±45/90] QI	Tr (GPa)
% [0]	100	0	50	50	50	25	
% [±φ]	0	100	50	50	40	50	
% [90]	0	0	0	0	10	25	
IM6/epoxy	0.874	0.762	0.676	0.504	0.518	0.337	232
IM7/977-3	0.877	0.762	0.675	0.504	0.519	0.337	218
T300/5208	0.877	0.765	0.678	0.504	0.518	0.337	206
IM7/MTM45	0.897	0.772	0.677	0.503	0.523	0.337	195
T800/Cytec	0.888	0.763	0.670	0.496	0.518	0.335	183
IM7/8552	0.884	0.766	0.676	0.501	0.519	0.336	180
T800S/3900	0.898	0.767	0.675	0.496	0.521	0.335	168
T300/F934	0.883	0.773	0.676	0.497	0.517	0.335	168
T700 C-Ply 64	0.866	0.758	0.674	0.500	0.514	0.337	163
AS4/H3501	0.852	0.750	0.672	0.503	0.512	0.338	162
T650/epoxy	0.866	0.757	0.672	0.498	0.514	0.337	160
T4708/MR60H	0.897	0.772	0.675	0.497	0.521	0.335	158
T700/2510	0.877	0.763	0.674	0.496	0.515	0.336	144
AS4/MTM45	0.889	0.770	0.676	0.496	0.518	0.335	144
T700 C-Ply 55	0.869	0.760	0.674	0.499	0.515	0.337	139
Mean	0.880	0.764	0.675	0.500	0.517	0.336	1
Coeff. Var. %	1.50	0.83	0.31	0.66	0.58	0.31	

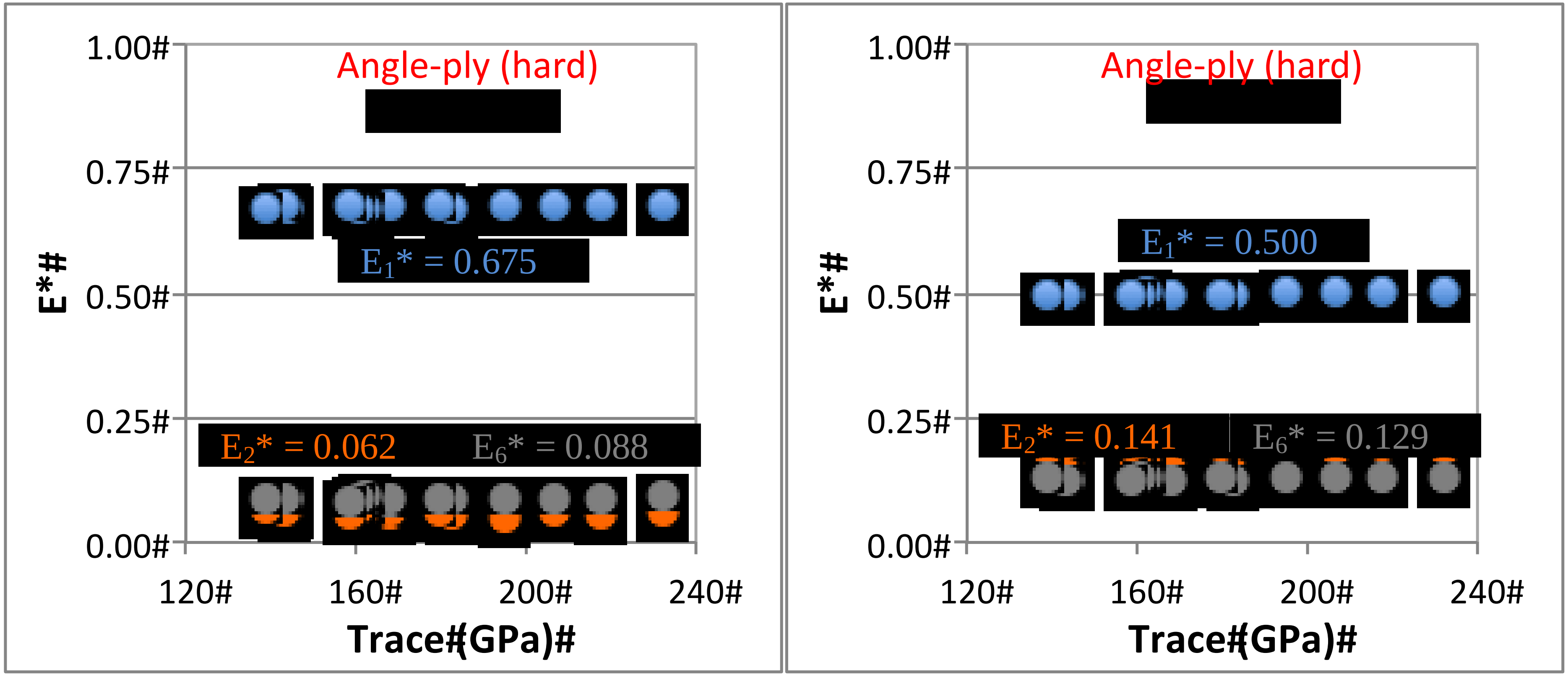
Problem needs to be solved only once for each laminate

Designing with Trace



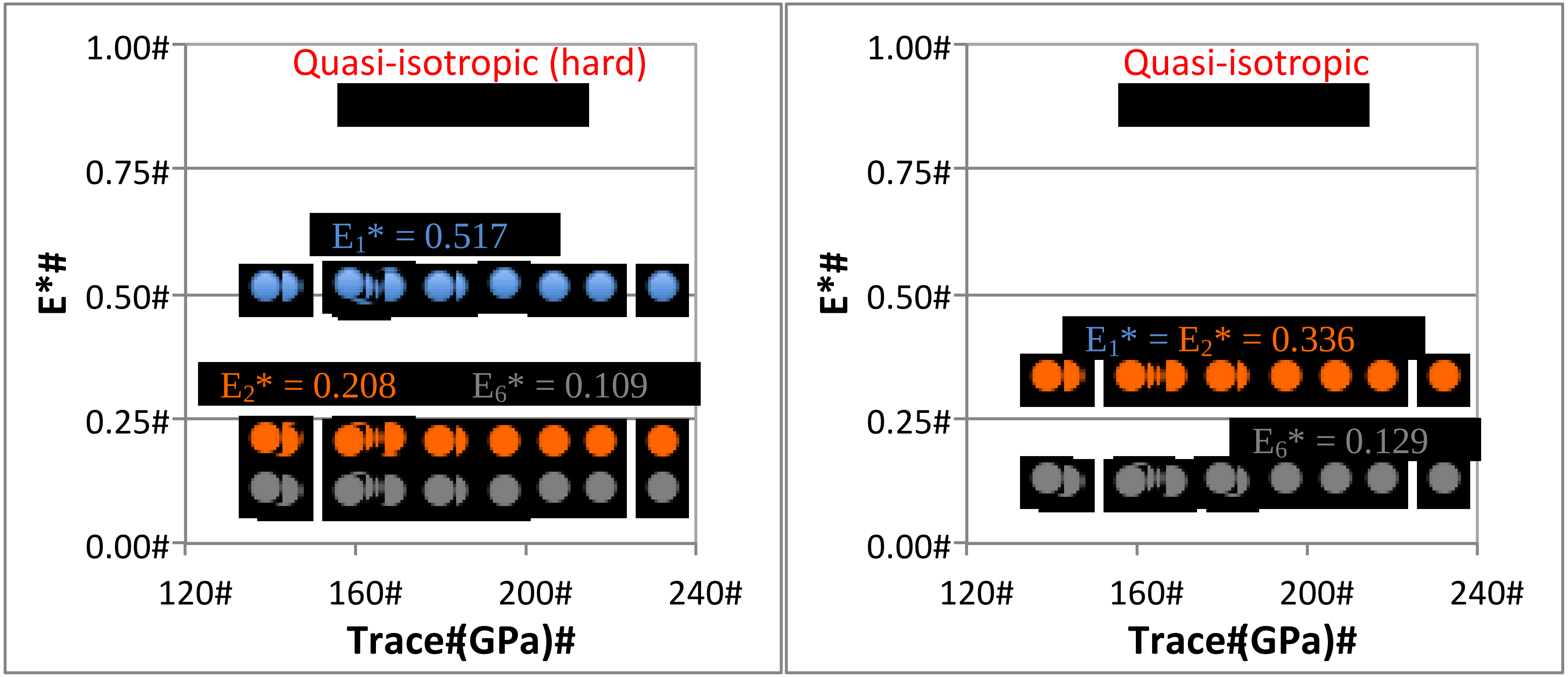
Laminates for 15 carbon/polymer materials

DESIGNING WITH TRACE



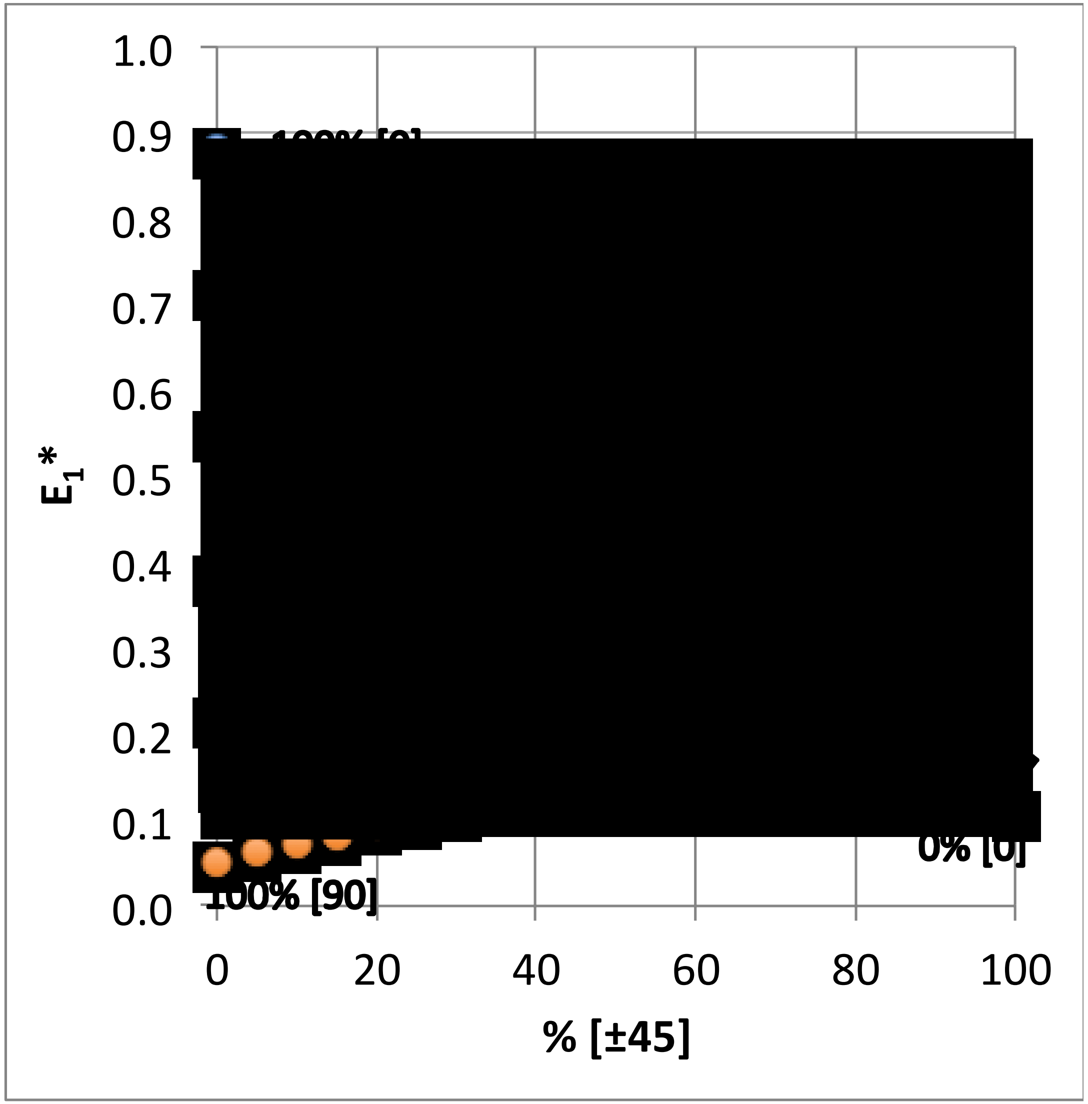
Laminates for 15 carbon/polymer materials

DESIGNING WITH TRACE



Laminates for 15 carbon/polymer materials

DESIGNING WITH TRACE

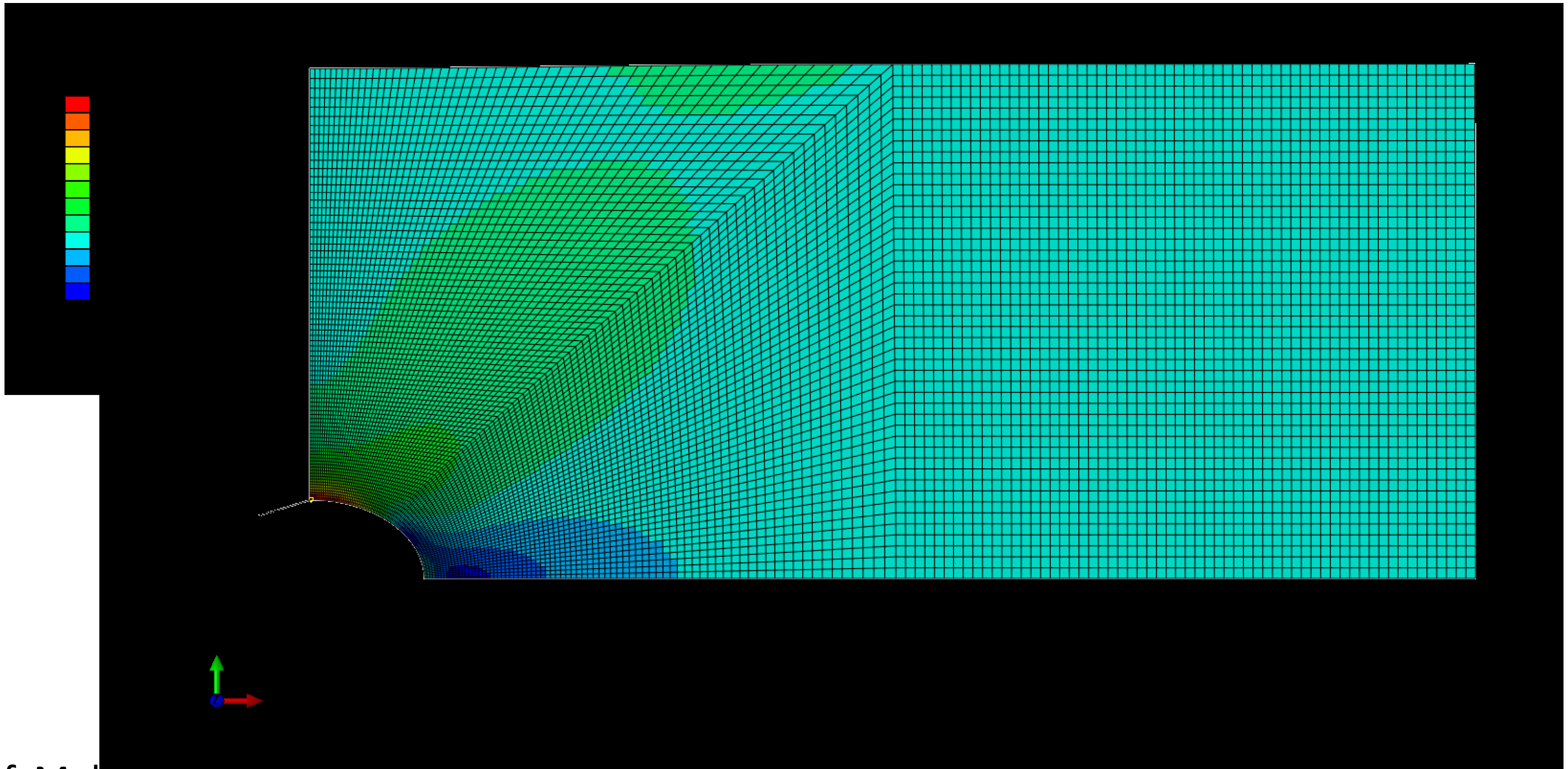


Carpet plot for the longitudinal Young's modulus of a [π/4] family using master ply properties.

DESIGNING W/ TRACE & UNIT CIRCLE

Open-Hole: FEA analysis (Abaqus®)

Contour plot of failure index (k) for AS4/H3501 $[0/\pm45/90]_{2S}$
Load {0.25, 0.0, 0.0}



Ref. Melo JDD, Bi J and Tsai SW. A novel invariant-based design approach to carbon fiber reinforced laminates. *Composite Structures* 159 (2017) 44–52.

DESIGNING W/ TRACE & UNIT CIRCLE

Open-Hole: FEA analysis (Abaqus®)

Failure indices (k) based on Unit Circle (U-C) and Tsai-Wu (T-W) failure criteria for open hole CFRP laminates under various load cases from FEA analyses

Load Case $\{N_1, N_2, N_6\}$ (MN/m)	Soft $[0/\pm45/90]_{2s}$ k		Hard $[0_5/\pm45/90]_s$ k		Soft $[0/\pm45_3/90]_s$ k	
	U-C	T-W	U-C	T-W	U-C	T-W
T700 C-Ply 55						
{0.25, 0.0, 0.0}	0.47	0.42	0.42	0.30	0.75	0.54
{0.25, 0.1, 0.0}	0.41	0.36	0.39	0.25	0.62	0.45
{0.25, −0.1, 0.0}	0.54	0.47	0.67	0.35	0.87	0.63
{−0.25, 0.0, 0.0}	0.66	0.68	0.55	0.49	0.92	0.93
AS4/H3501						
{0.25, 0.0, 0.0}	0.78	0.72	0.66	0.53	1.12	0.91
{0.25, 0.1, 0.0}	0.67	0.62	0.60	0.44	0.93	0.76
{0.25, −0.1, 0.0}	0.89	0.82	0.77	0.61	1.30	1.06
{−0.25, 0.0, 0.0}	0.78	0.78	0.66	0.57	1.12	1.07
T300/5208						
{0.25, 0.0, 0.0}	0.76	0.69	0.64	0.50	1.09	0.86
{0.25, 0.1, 0.0}	0.65	0.59	0.59	0.42	0.91	0.72
{0.25, −0.1, 0.0}	0.86	0.78	0.75	0.58	1.27	1.00
{−0.25, 0.0, 0.0}	0.76	0.77	0.64	0.55	1.09	1.08

In general, a larger failure index is obtained for U-C as compared to T-W

Ref. Melo JDD, Bi J and Tsai SW. A novel invariant-based design approach to carbon fiber reinforced laminates. *Composite Structures* 159 (2017) 44–52.

CONCLUSIONS

- Trace and unit circle failure criterion can greatly simplify design and testing of CFRP laminates
  - Design for stiffness can be carried out using master ply properties: Material properties are added later
  - The optimized solution using master ply applies to laminates made of any UD CFRP
  - Trace is the only material property needed as the scaling factor for the determination of strains
  - Testing for trace and unit circle require fewer properties to be measured
- Failure analyses of open hole CFRP laminates indicated that the unit circle is more conservative than Tsai-Wu

THANK YOU!

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