Modal Analysis of Separated Nozzle Flow

Ragnar Lárusson

Chalmers University of Technology Department of Applied Mechanics Division of Fluid Mechanics Gothenburg, Sweden

ragnar.larusson@chalmers.se



Partners



Space Exploration and Exploitation



The Convergent-Divergent Nozzle (CD Nozzle)



CD Nozzle - Small Example



CD Nozzle - Big Example



Real Rocket Engine Jet



Cold Flow Supersonic Jet



Figure: Experimental nozzle at DLR Lampholdshausen. From Génin and Stark 2016

Definition of a CD Nozzle



Separated Nozzle Flow



Separated Nozzle Flow



Separated Nozzle Flow - Free Shock Separation



Separated Nozzle Flow - Schlieren Photograph



Nozzle Side-Loads

- Asymmetrical separation inside a supersonic nozzle causes unwanted lateral forces, or side-loads, acting on the nozzle structure
- Semi-empirical side-load models provide only moderate accuracy

Nozzle Side-Loads

- Computational Fluid Dynamics (CFD) provides a way to directly simulate flow processes that are responsible for the side-loads
- Modal Decomposition of computational and experimental data can provide information about the nature of flow instabilities and their fluctuation frequencies

Modal Decomposition

- Identifies fluctuating modes of a unstable flow field, similar to eigenfrequency analysis in structural mechanics.
- In the current project two different modal decomposition algorithms have been applied for the first time to separated nozzle flow. These are the Dynamic Mode Decomposition and the Arnoldi algorithms respectively.

Modal Decomposition Example



Ragnar Lárusson - Chalmers

Modal Decomposition Example



Ragnar Lárusson - Chalmers

Computational Fluid Dynamics

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho} \tilde{u}_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial \overline{\rho} \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} \left(\overline{\rho} \tilde{u}_i \tilde{u}_j + \overline{\rho} \delta_{ij} - \tau_{ij} \right) = 0$$
⁽²⁾

$$\frac{\partial \overline{\rho} \tilde{e}_0}{\partial t} + \frac{\partial}{\partial x_j} \left(\overline{\rho} \tilde{e}_0 \tilde{u}_j + \overline{p} \tilde{u}_j - C_p \left(\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{\partial \tilde{T}}{\partial x_j} \right) - \tilde{u}_i \tau_{ij} \right) = 0 \tag{3}$$

$$\tau_{ij} = (\mu + \mu_t) \left(2\tilde{S}_{ij} - \frac{2}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \bar{\rho} \tilde{k} \delta_{ij} \tag{4}$$

$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)$$
(5)

$$\frac{\partial \overline{\rho} \tilde{k}}{\partial t} + \frac{\partial}{\partial x_j} \left(\overline{\rho} \tilde{k} \tilde{u}_j - \left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial \tilde{k}}{\partial x_j} \right) = P_k - \overline{\rho} \tilde{\varepsilon}$$
(6)

$$\frac{\partial \overline{\rho}\tilde{\varepsilon}}{\partial t} + \frac{\partial}{\partial x_j} \left(\overline{\rho}\tilde{\varepsilon}\tilde{u}_j - \left(\mu + \frac{\mu_t}{\sigma_{\varepsilon}}\right) \frac{\partial \tilde{\varepsilon}}{\partial x_j} \right) = (C_{\varepsilon 1}P_k - C_{\varepsilon 2}\overline{\rho}\tilde{\varepsilon})\frac{\tilde{\varepsilon}}{\tilde{k}}$$
(7)

$$P_{k} = \left(\mu_{t} \left(2\tilde{S}_{ij} - \frac{2}{3}\frac{\partial\tilde{u}_{k}}{\partial x_{k}}\delta_{ij}\right) - \frac{2}{3}\overline{\rho}\tilde{k}\delta_{ij}\right)\frac{\partial\tilde{u}_{i}}{\partial x_{j}}$$

$$\tag{8}$$

$$\mu_t = \min\left(C_{\mu}\overline{\rho}\frac{\tilde{k}^2}{\tilde{\varepsilon}}, \frac{C_r\overline{\rho}\tilde{k}}{\sqrt{\tilde{S}_{ij}\tilde{S}_{ij}}}\right) \tag{9}$$

G3D::FLOW

An explicit compressible Navier-Stokes equation solver.

Jointly developed at GKN Aerospace Sweden AB and Chalmers University of Technology.

Separated Nozzle Flow - Steady State Flow Field



Separated Nozzle Flow - Arnoldi Mode Example



Transonic Resonance - DMD Mode Exmaple



High Cost vs. Low Cost Modal Decomposition



High Fidelity CFD



High Fidelity CFD - Density Gradient Magnitude



Ragnar Lárusson - Chalmers

High Fidelity CFD - Vorticity Magnitude Iso-surface



Ragnar Lárusson - Chalmers

High Fidelity CFD - Simulated vs. Measured Side-Load



Figure: Raw and corrected measured σ_M side-load vs. simulated values.

High Fidelity CFD - DMD Mode Example







CHALMERS

