Multiple Damage Detection in Plate Structure using Piezoelectric Array Sensors and 2D-MUSIC Spectrum

S. V. Farias[1], O. Saotome[1], E. G. O. Nóbrega[2], I. Sander[3], I. Söderquist[4] and D. S. Loubach[2]

[1]Electronics Engineering Division, Aeronautics Institute of Technology-ITA, São José dos Campos, Brazil
(Email: *sertone@gmail.com, osaotome@ita.br*)
[2]Advanced Computing, Control & Embedded Systems Lab / FEM, University of Campinas – UNICAMP, Campinas, Brazil
(Email: *egon@fem.unicamp.br, dloubach@fem.unicamp.br*)
[3]Electronics and Embedded Systems Department, KTH Royal Institute of Technology, Stockholm, Sweden
(Email: *ingo@kth.se*)
[4]HMI & Avionics Department, Saab AB, Linköping, Sweden
(Email: *ingemar.soderquist@saabgroup.com*)

INTRODUCTION

The damage detection and localization is an important issue in a structure health monitoring (SHM) system. Recently, the crescent use of composite materials in aircraft structures has augmented the interest in SHM due to their susceptibility to damages resulting the impacts (Zhong, et al., 2014). Normally, a SHM solution has an embedded sensor-actuator network in the structure in order to provide an in-service assessment.

The guided Lamb wave propagation is an active SHM research area and a promise tool for real applications, due to their long propagation capability and sensitivity to different kinds of damage. The piezoelectric transducers have been used to transmit and receive Lamb waves (Giurgiutiu, 2005).

Several sensor array signal processing based methods have been proposed. The array method consists of transducer groups to scan large areas of the structure from a single location. The sensor array methods presented in literature are phased array (Rucka & Wilde, 2006) (Kudela, et al., 2007) and high-resolution spectral estimation approach.

Among the high-resolution spectral estimation methods, the MUSIC (Multiple Signal Classification) is a promising tool because of its capability to detect multiple damages under low signal-to-noise ratio and directional scanning ability (Zhong, et al., 2014) (Bao, et al., 2015). A MUSIC algorithm and uniform circular array to detect the direction-of-arrival (DOA) of Lamb wave source on an aluminum plate is presented in (Engholm & Stepinski, 2011). A 2D-MUSIC algorithm and uniform linear array to detect damages in a plate of composite material is presented in (Li, et al., 2014) . The algorithm is suitable for near-field condition. A MUSIC algorithm and uniform linear array to determine the impact location is presented in (Bao, et al., 2015).

In this paper, a 2D-MUSIC algorithm using two uniform linear array (ULA) to detect damages through the estimation of the direction-of-arrival (DOA) and direction-of-departure (DOD) is presented. The similar algorithm has been used in MIMO (Multiple-In-Multiple-Out) radar (Liu & Liao, 2011). There is a transmitter array and a receiver array. The distance between the arrays is known, therefore the polar coordinate of the damages is calculated by a triangulation process. Simulations in MATLAB were performed in order to test the proposed method. The obtained results evidence a satisfactory real time detection of multiple damages in simulated plate structure.

MULTIPLE DAMAGE DETECTION ALGORITHM

Mathematical Model

The damage detection system is composed of a transmitter array and a receiver array. The system is depicted in Fig. 1. The transmitter array has *M* elements and the distance between two elements is *dt*, while the receiver array has *N* elements and the distance between two elements is *dr*. The distance between the arrays is *Da*. The signal emitted by the transmitter array is reflected by each damage and received in the receiver array. Fig. 1 shows the signal path for the *p*th damage. The angle θ_p is the direction-of-departure (DOD) angle related to the *p*th damage and α_p is the direction-of-arrival (DOA) angle related to the *p*th damage.

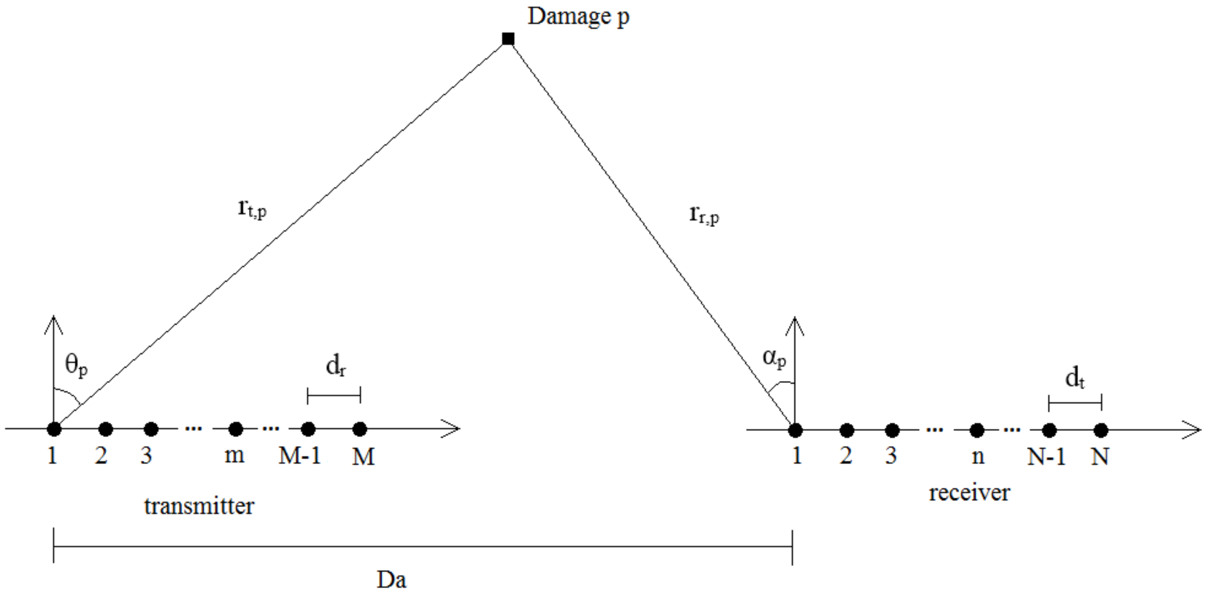

Fig. 1 – Arrangement to damage detection

The signal emitted by each transducer in the transmitter array is given by the Eq. (1).

$$s(t) = u(t)e^{j\omega t} \tag{1}$$

where ω is the carrier frequency and *u(t)* the window signal.
The signal transmitted by the transducer *m*, reflected by the damage *p* and received by the transducer *n* is represented by Eqs. (2) to (5) (Giurgiutiu, 2005) (Li, et al., 2014).

$$s_{m,n,p}(t) = b_p(t) \cdot a_{R,n}(\alpha_p) \cdot a_{T,m}(\theta_p) \tag{2}$$

$$b_p(t) = \frac{u\left(t - \frac{r_{t,p} + r_{r,p}}{c}\right)}{\sqrt{r_{t,p} \cdot r_{r,p}}} e^{j[\omega t - k(r_{t,p} + r_{r,p})]} \tag{3}$$

$$a_{T,m}(\theta_p) = e^{jk(m-1)d_t \sin(\theta_p)} \tag{4}$$

$$a_{R,n}(\alpha_p) = e^{jk(n-1)d_r \sin(\alpha_p)} \tag{5}$$

where c is the wave speed and k is the wavenumber.
Considering P damages, the signal transmitted by the transducer m and received by the transducer n is given by the Eq. (6).

$$s_{m,n}(t) = \sum_{p=1}^{P} a_{R,n}(\alpha_p) \cdot a_{T,m}(\theta_p) \cdot b_p(t) + u_{m,n}(t) \tag{6}$$

where $u_{m,n}(t)$ is the background noise (Li, et al., 2014).
Writing Eq. (6) in a matrix form Eqs. (7) to (12) are obtained (Zhang, et al., 2010).

$$\boldsymbol{s}(t) = A \cdot \boldsymbol{b}(t) + \boldsymbol{u}(t) \tag{7}$$

$$\boldsymbol{s}(t) = \left[s_{1,1}\ s_{1,2}\ \dots\ s_{1,M}\ s_{2,1}\ s_{2,2}\ \dots s_{2,M}\ \dots s_{N,1}\ s_{N,2}\ \dots s_{N,M}\right]^T \tag{8}$$

$$\boldsymbol{a_R}(\alpha_p) = \left[a_{R,1}(\alpha_p)\ a_{R,2}(\alpha_p) \dots\ a_{R,N}(\alpha_p)\right]^T \tag{9}$$

$$\boldsymbol{a_T}(\theta_p) = \left[a_{T,1}(\theta_p)\ a_{T,2}(\theta_p) \dots\ a_{T,M}(\theta_p)\right]^T \tag{10}$$

$$A = [\boldsymbol{a_R}(\alpha_1) \otimes \boldsymbol{a_T}(\theta_1)\ \ \boldsymbol{a_R}(\alpha_2) \otimes \boldsymbol{a_T}(\theta_2)\ \dots\ \boldsymbol{a_R}(\alpha_P) \otimes \boldsymbol{a_T}(\theta_P)] \tag{11}$$

$$\boldsymbol{u}(t) = \left[u_{1,1}\ u_{1,2}\ \dots\ u_{1,M}\ u_{2,1}\ u_{2,2}\ \dots u_{2,M}\ \dots u_{N,1}\ u_{N,2}\ \dots u_{N,M}\right]^T \tag{12}$$

where $\otimes$ is the Kronecker product and $(.)^T$ denotes transpose.
The vector **s**(t) has dimension $N \cdot M$ x 1, the matrix A has dimension $N \cdot M$ x P, the vector **b**(t) has dimension P x 1 and the vector **u**(t) has dimension $N \cdot M$ x 1.

2D-MUSIC Algorithm

The proposed 2D-MUSIC (Multiple Signal Classification) is an algorithm that provides asymptotically unbiased estimates of the number of damages (estimating the rank of the covariance matrix of **s**(t)) and the DOA and DOD of damages (peaks in the MUSIC spectrum) (Schmidt, 1986).
The covariance matrix of s(t) is calculated by Eq. (13) considering L snapshots.

$$R_s = \frac{1}{L} \sum_{l=1}^{L} \boldsymbol{s}(t_l)\, \boldsymbol{s}(t_l)^H \tag{13}$$

where $(.)^H$ denotes Hermitian.
Eq. (14) represents the matrix Rs using its eigen-decomposition (Zhang, et al., 2010). E_s is a matrix composed of the eingenvectors related to the P largest eigenvalues of R; E_s is a matrix composed of the eingenvectors related to the P largest eigenvalues.

$$R_s = E_s D_s {E_s}^H + E_n D_n {E_n}^H \tag{14}$$

where: D_s is a diagonal matrix of order P composed of the P largest eigenvalues of R_s ; D_n is a diagonal matrix of order $N \cdot M - P$ composed of the $N \cdot M - P$ smallest eigenvalues of R_s ; E_s is a matrix composed of the eingenvectors related to the P largest eigenvalues of R; E_n is a matrix composed of the eingenvectors related to the $N \cdot M - P$ smallest eigenvalues of R_s.

The 2D-MUSIC spectrum function is given by Eqs. (15) to (17) (Zhang, et al., 2010).

$$P_{MU}(\alpha,\theta) = \frac{1}{[\boldsymbol{a_R}(\alpha)\otimes\boldsymbol{a_T}(\theta)]^H(E_nE_n{}^H)[\boldsymbol{a_R}(\alpha)\otimes\boldsymbol{a_T}(\theta)]} \tag{15}$$

where:

$$\boldsymbol{a_R}(\alpha) = \left[a_{R,1}(\alpha)\ a_{R,2}(\alpha) \dots\ a_{R,N}(\alpha)\right]^T \tag{16}$$

$$\boldsymbol{a_T}(\theta) = \left[a_{T,1}(\theta)\ a_{T,2}(\theta) \dots\ a_{T,M}(\theta)\right]^T \tag{17}$$

The P largest peaks of $P_{MU}(\alpha,\theta)$ are the estimate of the DOA and DOD of the damages. The damage location is function of DOA and DOD angles. Using the triangulation process, the distance of the *p*th damage to the receiver array is given by Eq. (18).

$$d_p = \frac{Da \cdot \cos(\alpha_p)}{\sin(\theta_p + \alpha_p)} \tag{18}$$

Therefore, the position of the *p*th damage is (d_p, θ_p).

RESULTS

The proposed 2D-MUSIC algorithm was validated by using MATLAB simulation of the Lamb wave propagation through an aluminum plate (Fig. 2). The simulated plate has 2 m width, 2 m height and 5 mm thickness. The transmitter array with 12 piezoelectric transducers was positioned in the center of the plate and the receiver array was placed at a distance of 0.5 m from the transmitter one. The arrays are represented by a yellow color in Fig. 2. The distance between the transducers is 7 mm in both arrays.

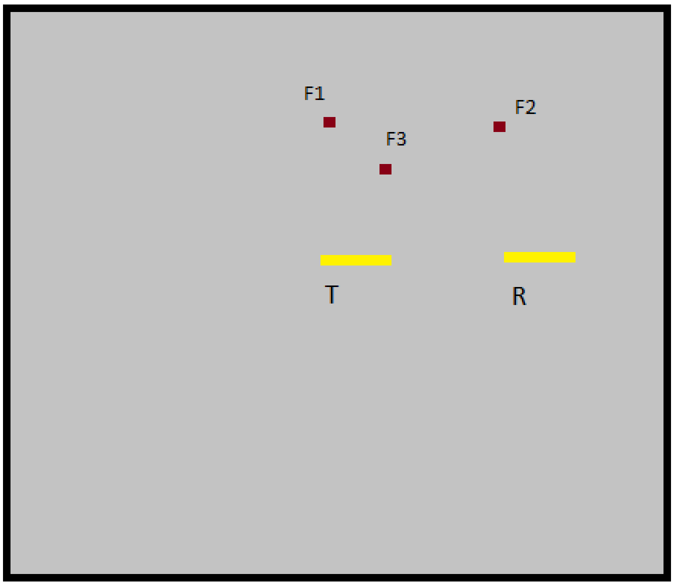

Fig. 2 – Aluminum plate with damages

Three damages F1 (0.6 m, 0°), F2 (0.7 m, 40°) and F3 (0.4 m, 30°) were simulated. The angles were calculated in relation to the receiver array and correspond the DOD of the damages. The triangulation geometry is described in Fig. 3 and the angles calculation uses the Eq. (18).

Therefore, the damage positions in function of DOA and DOD are: F1 (39.8º, 0º), F2 (5.33º, 40º) and F3 (40.98º, 30º).

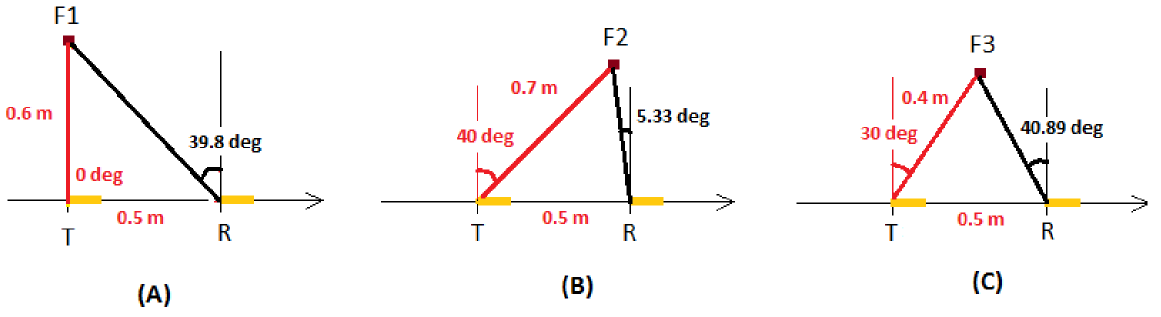

Fig. 3 – Triangulation process

The signal was emitted with a carrier frequency of 150 kHz, the wave propagates with phase velocity of 5000 m/s and the signal-to-noise ratio is 7 dB.
The 2D-MUSIC spectrum is depicted in figure 4. Theta is DOD angle and alpha is the DOA angle. The three damages were detected close to the correct DOD and DOA positions.

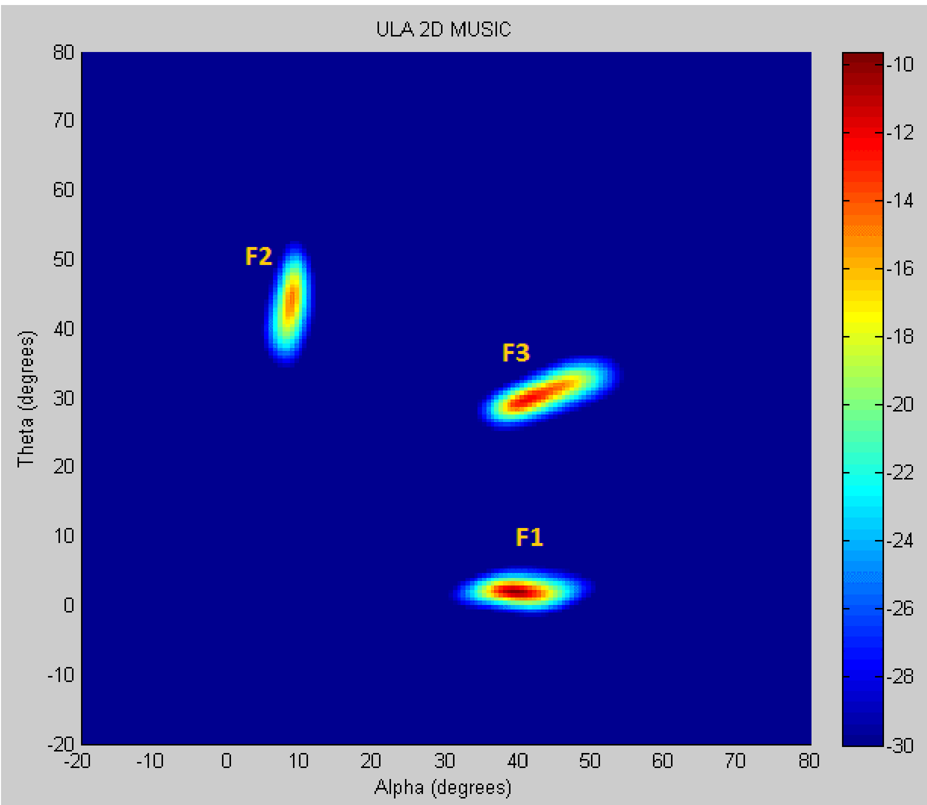

Fig. 4 – 2D-MUSIC Spectrum in dB

CONCLUSION

A 2D-MUSIC algorithm to multiple damages detection was proposed. The algorithm works using a transmitter array and receiver array. The damages position, in the conventional 2D-MUSIC spectrum applied to radar MIMO applications, is related to DOA and DOD information, so the triangulation method was used in order to represent the damages location in polar coordinates.
The simulation results are presented, in order to validate the proposed algorithm.

REFERENCES

Bao, Q., Qiu, L. & Zhong, Y., 2015. *Research on single frequency component based MUSIC algorithm for impact localization.* Xi'an, China, International Conference on Noise and Fluctuations.

Engholm, M. & Stepinski, T., 2011. Direction of arrival estimation of Lamb waves using circular arrays. *Structural Health Monitoring,* September, Volume 10, pp. 467-480.

Giurgiutiu, V., 2005. *Structural health monitoring with piezoelectric wafer active sensors.* France, s.n.

Kudela, P., Ostachowicz, W. & Zak, A., 2007. Damage detection in composite plates with embedded PZT transducers. *Mechanical Systems and Signal Processing*, 2 August, pp. 1327-1335.

Li, F., Peng, H. & Meng, G., 2014. Quantitative damage image construction in plate structures using a circular PZT array and lamb waves. *Sensors and Actuators*, 14 April, pp. 66-73.

Liu, X. & Liao, G., 2011. *Reduced-dimensional Angle Estimation in Bistatic MIMO Radar System.* Chengdu, China, IEEE CIE International Conference on Radar.

Rucka, M. & Wilde, K., 2006. Application of continuous wavelet transform in vibration based damage detection method for beams and plates. *Journal of Sound and Vibration,* April, 207(3-5), pp. 536-550.

Schmidt, R. O., 1986. Multiple Emitter Location and Signal Parameter Estimation. *IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION*, March, pp. 276-280.

Zhang, X., Xu, L. & Xu, D., 2010. Direction of Departure (DOD) and Direction of Arrival (DOA) Estimation in MIMO Radar with Reduced-Dimension MUSIC. *IEEE COMMUNICATIONS LETTERS,* December, Volume 14, pp. 1161-1163.

Zhong, Y., Yuan, S. & Qiu, L., 2014. Multiple damage detection on aircraft composite structures using near-field MUSIC algorithm. *Sensors and Actuators A: Physical,* April.pp. 234-244.