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# Modelling of Transonic and Supersonic Aerodynamics for Conceptual Design and Flight Simulation

# Motivation

- Model should be efficient for flight simulation.
- To establish requirements System of systems simulations need to be done in the conceptual phase.
- In conceptual design modelling is aimed to predict the behavior and performance of the finished product. As such they should not necessarily show the exact behaviour for the aircraft at the sketchy level of conceptual design.
- Model with a minimum of parameters.
- Model that can be flexible enough to be fitted against a high-fidelity model

# Non-Linear Aerodynamic Model

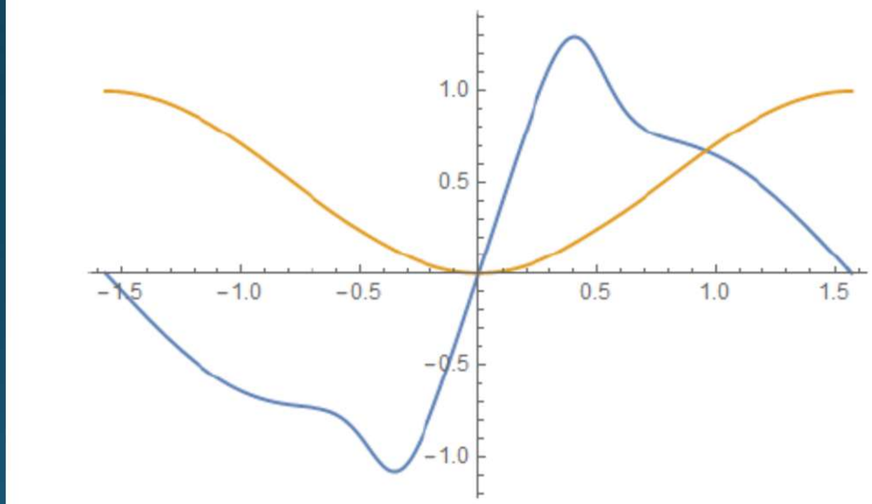



*Figure 1: Non-linear aerodynamic model. Lift coefficient $C_L$ and induced drag $C_{di}$ as a function of $\alpha$.*

# The Logistic Function

$$f_M = \frac{1}{1+e^{-8\frac{M-(1-\delta_M/2)}{\delta_M}}} \quad (3)$$

Here:

$$\delta_M = 1 - M_{crit} \quad (4)$$

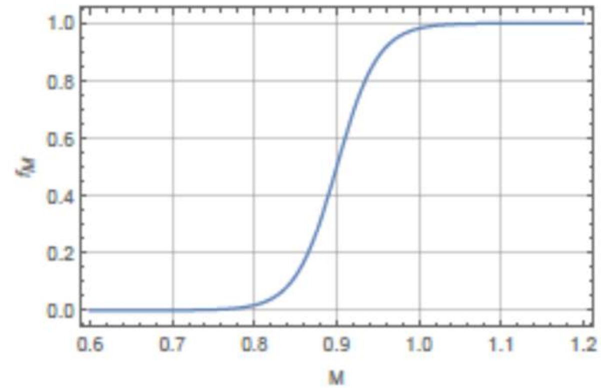



*Figure 2: The logistic function for $\delta_M = 0.2$. Hence for this example $M_{crit} = 0.8$.*

# Aerodynamic Drag Estimation

$$C_{dw} = f_M C_{dw0} \frac{k_{dw}}{(((M - k_{dwm})^2 - 1)^2 + k_{dw}^4)^{1/4}} \quad (5)$$

Sears-Hack body

$$C_{dw,SH} = 9\pi \frac{S_{max}}{2L^2} \quad (6)$$

$$C_{dw,SH} = 9\pi \frac{S_{max}}{2L^2} \left( \frac{S_{max}}{S_{ref}} \right) \quad (7)$$

$$C_{dw} = E_{wd} C_{dw,SH} \quad (8)$$

Fudge factor to represent real aircraft

With $C_{dw0} = 0.0264$, $k_{dwm} = 0.05$, $k_{dw} = 0.5$ and $\delta M = 0.2$ the function in Fig. 3 is obtained.

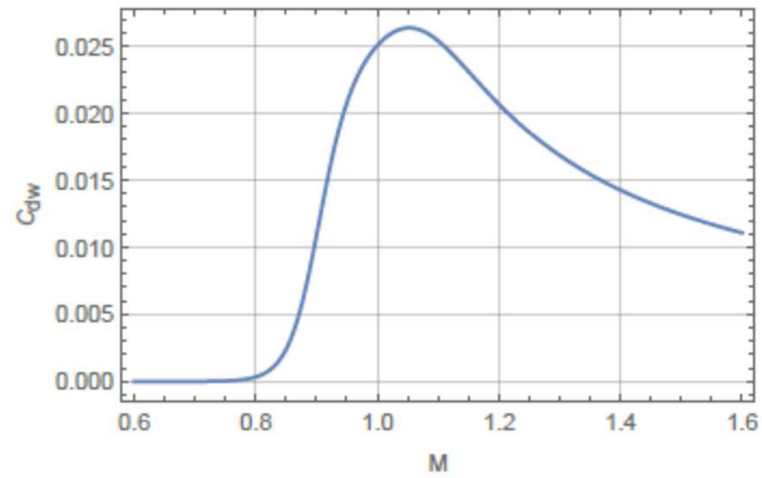



*Figure 3: Wave drag coefficient as a function of Mach number, using Eq. 5*

# Lift Coefficient

## Subsonic region

$$C_{L\alpha,sub} = \frac{C_{L\alpha,0}}{\beta} \qquad (9)$$

$$\beta = \sqrt{1 - M^2} \qquad (10)$$

## Supersonic region

$$C_{L\alpha,sub} = \frac{4}{\beta} \qquad (11)$$

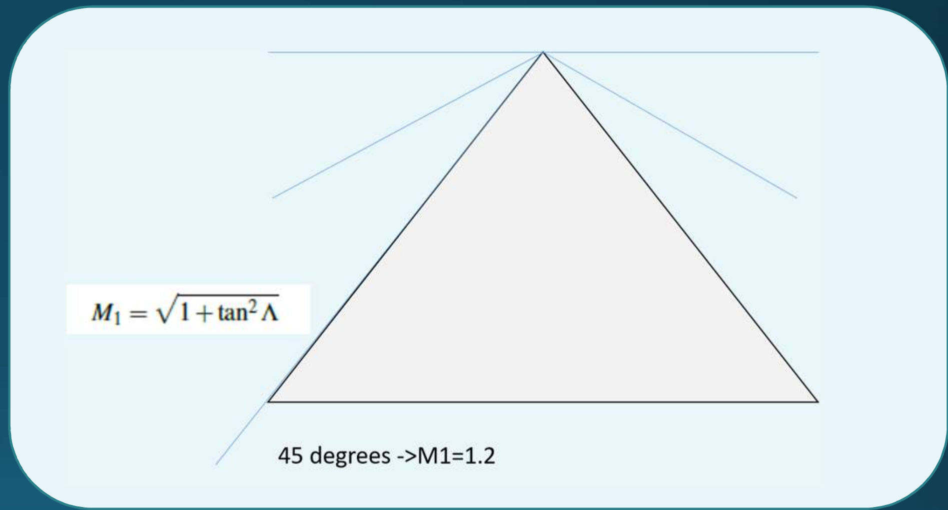
$$\beta = \sqrt{M^2 - 1} \qquad (12)$$

$$S_0 = S_{wb}/S_{ref} \qquad (13)$$

$$C_{L\alpha,sup} = S_0 \frac{4}{\beta} \qquad (14)$$

$$\beta = ((M^2 - 1)^2)^{1/4} \qquad (15)$$

$$\beta = ((M^2 - 1)^2 + \varepsilon_M^4)^{1/4} \qquad (16)$$




$$M_1 = \sqrt{1 + \tan^2 \Lambda} \qquad (17)$$

# Lift Coefficient

## Transonic region

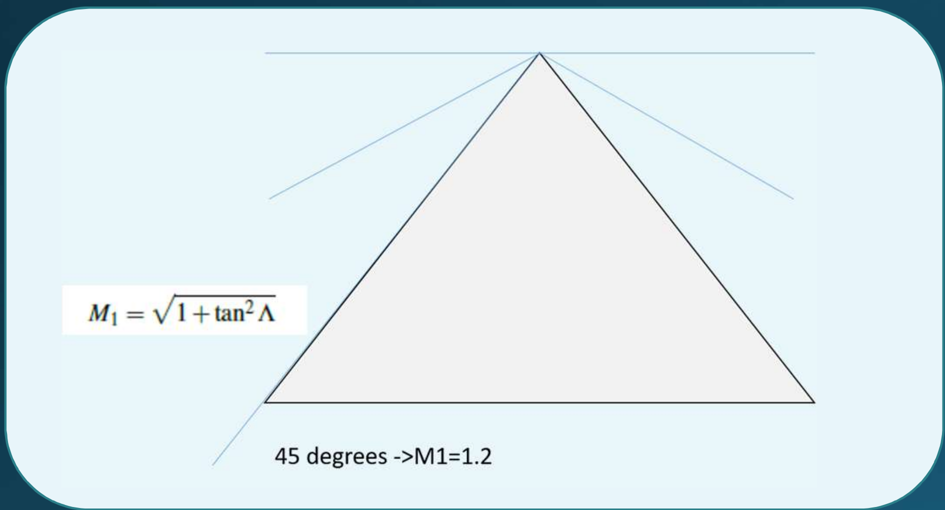



*Figure 4: Mach cone touching the leading edge of a delta wing.*

$$M_1 = \sqrt{1+\tan^2\Lambda} \tag{17}$$

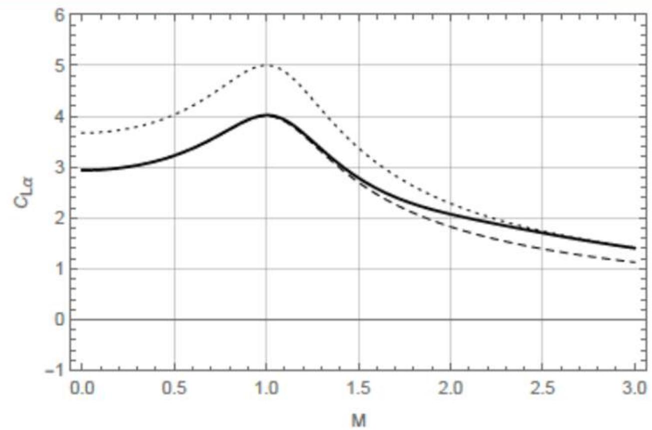



*Figure 5: The lift coefficient with respect to α. The dotted line is without leading edge suction and the dashed line is with leading edge suction. The solid line is a blend of of both for a high Mach cone angle*

# Moment Coefficient and Lift Dependent Drag Coefficient

Aerodynamic centre is moving from aprox. quarter cord to aprox. half cord

$$C_m = C_{m0} - C_L \frac{MAC}{4} f_M \qquad (21)$$

Lift Dependent Drag Coefficient

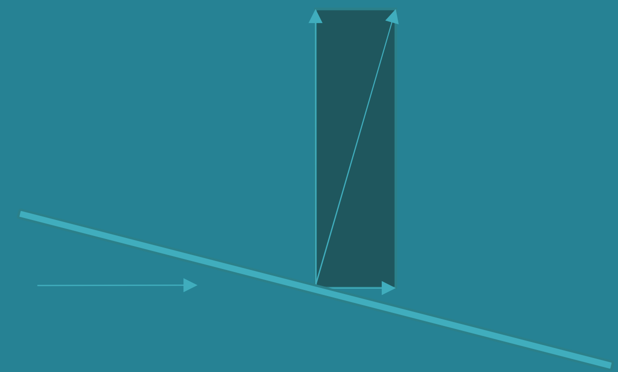
$$C_{Di} = \frac{C_L^2}{\pi e A} \qquad (22)$$

This can be rewritten as:

$$C_{Di} = \frac{C_{L\alpha}^2}{\pi e A}\alpha^2 = C_{Di\alpha^2,sub}\alpha^2 \qquad (23)$$

For a straight wing in supersonic flow there is no leading edge suction. This means that the lift dependent drag can be found from the lift from trigonometric relations:

$$C_{Di,sup} = C_L \tan(\alpha) \approx C_{L,sup}\alpha = C_{L\alpha,sup}\alpha^2 = C_{Di\alpha^2,sup}\alpha^2 \qquad (24)$$



No leading edge suction

# Moment Coefficient and Lift Dependent Drag Coefficient

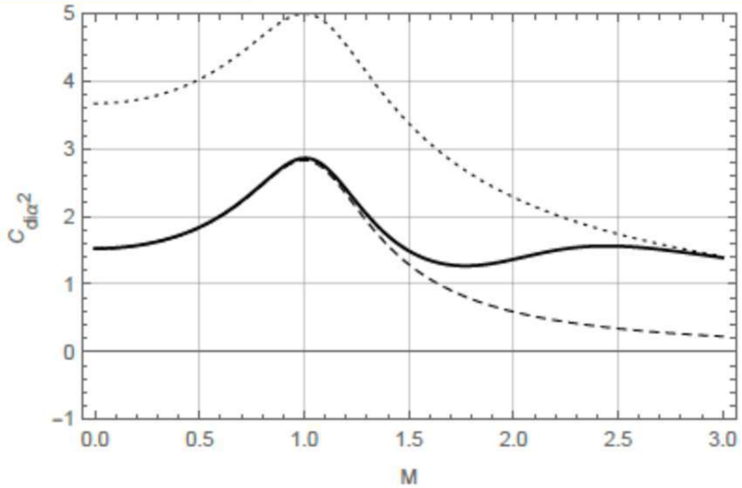



*Figure 6: The coefficient for induced drag with respect to $\alpha^2$. The dotted line is without leading edge suction and the dashed line is with leading edge suction. The solid line is a blend of both for a high Mach cone angle*

In the firmly supersonic regime

$$C_{Di,sup} = S_0 \frac{4}{\beta} \alpha^2 \quad (25)$$

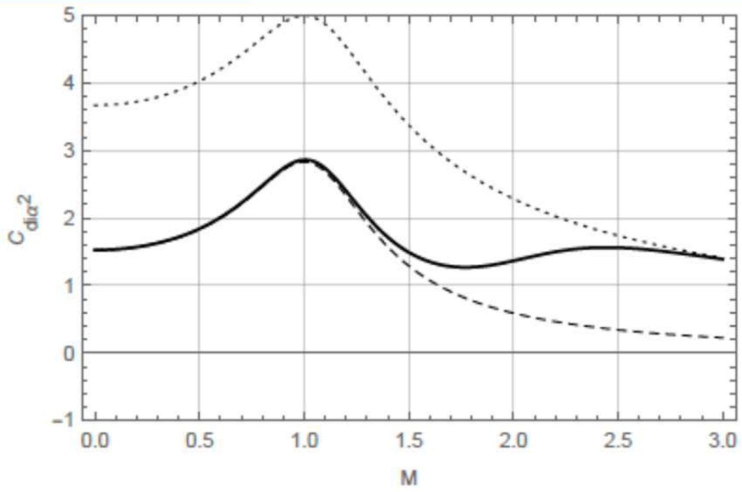
$$e_{sup} = \frac{C_{L,sup}^2}{\pi A C_{Di,sup}} = \frac{C_{L\alpha,sup}^2}{\pi A S_0 \frac{4}{\beta}} \quad (26)$$

$$e = e_{sub}(1 - f_{ML}) + e_{sup} f_{ML} \quad (27)$$

Then use in the whole regime

$$C_{Di} = \frac{C_L^2}{\pi e A} \quad (22)$$

# Moment Coefficient and Lift Dependent Drag Coefficient

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# Comparison of Contribution to Drag

Example: F-16 flying at 10000m with a weight of 12000kg

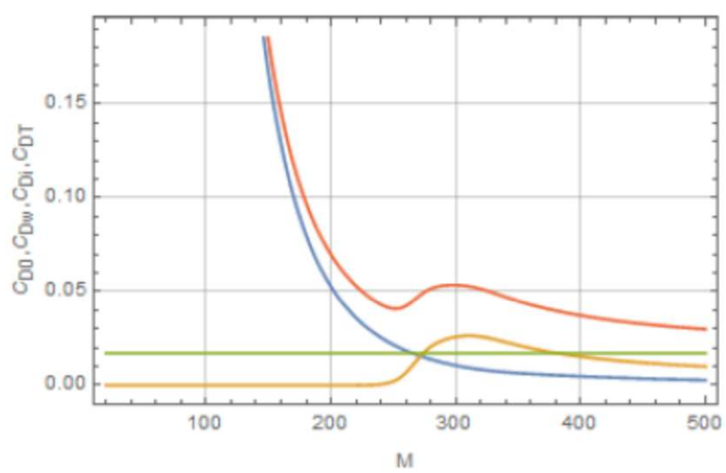



*Figure 7: The contributions of parasitic, the wave drag and the induced drag as well as the total drag.*

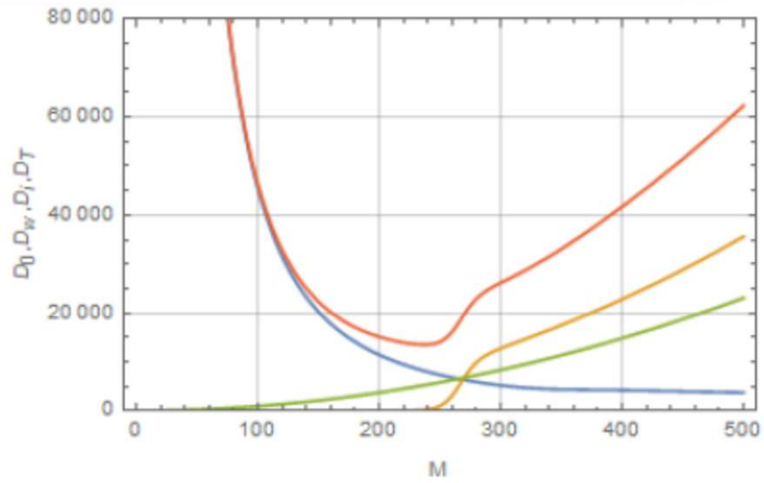
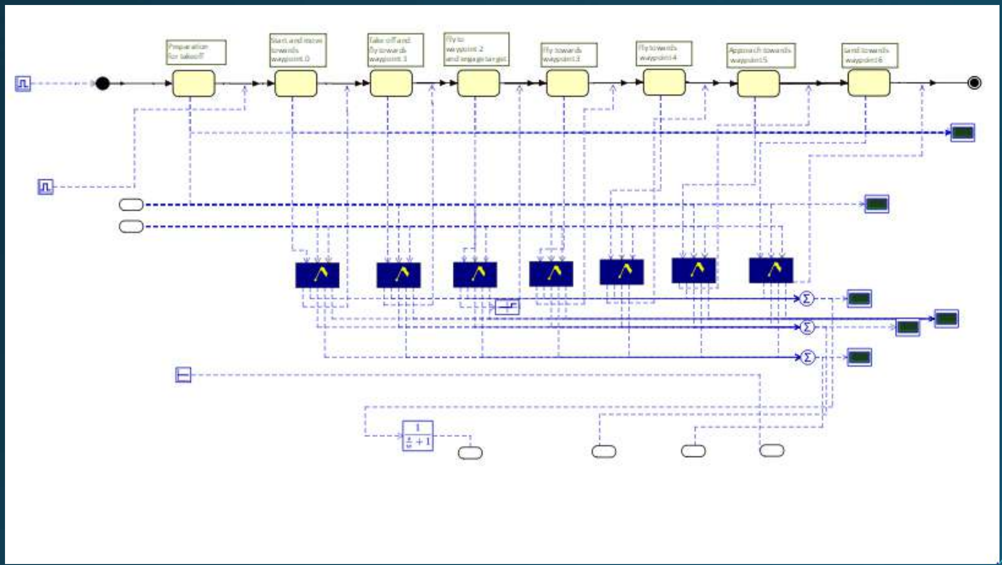
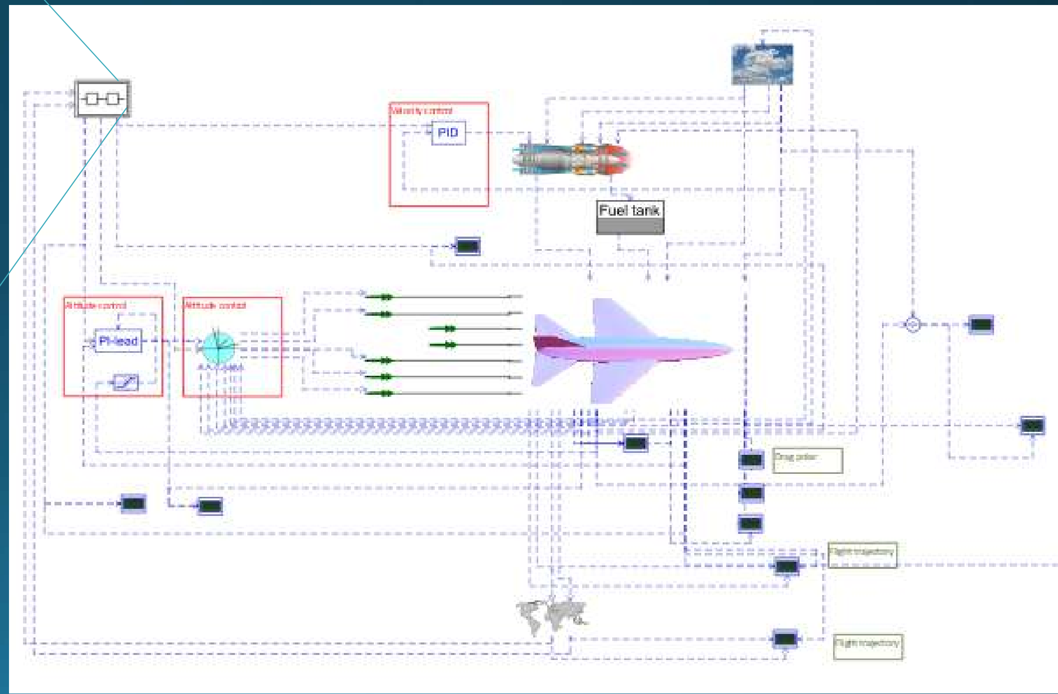



*Figure 8: The contributions of parasitic, the wave drag and the induced drag as well as the total drag.*

# System Simulation in Hopsan



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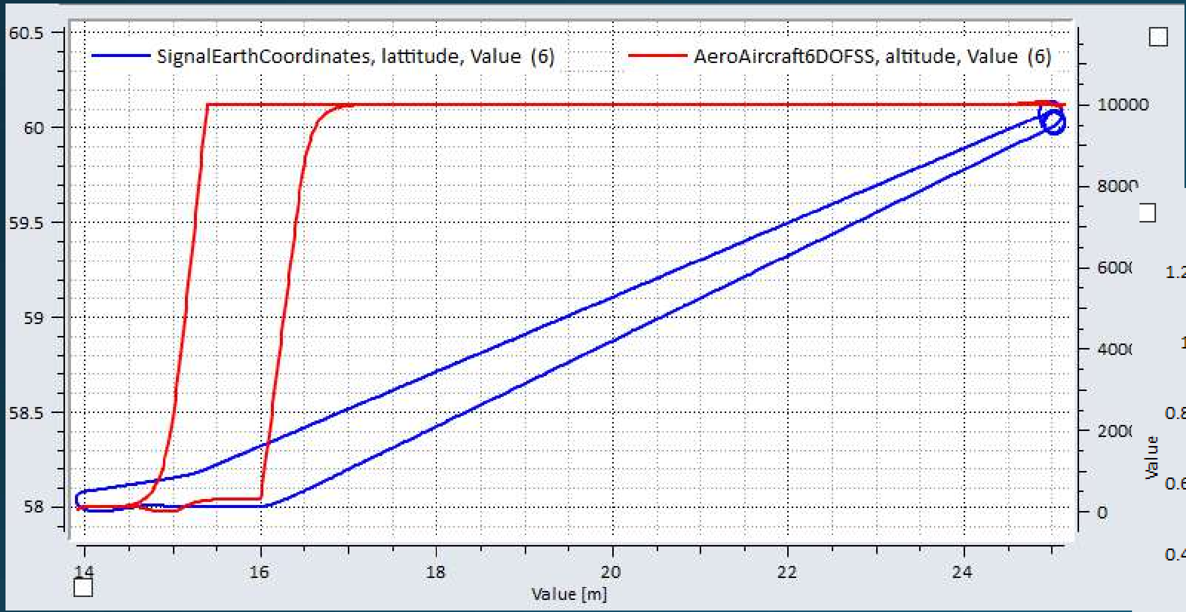
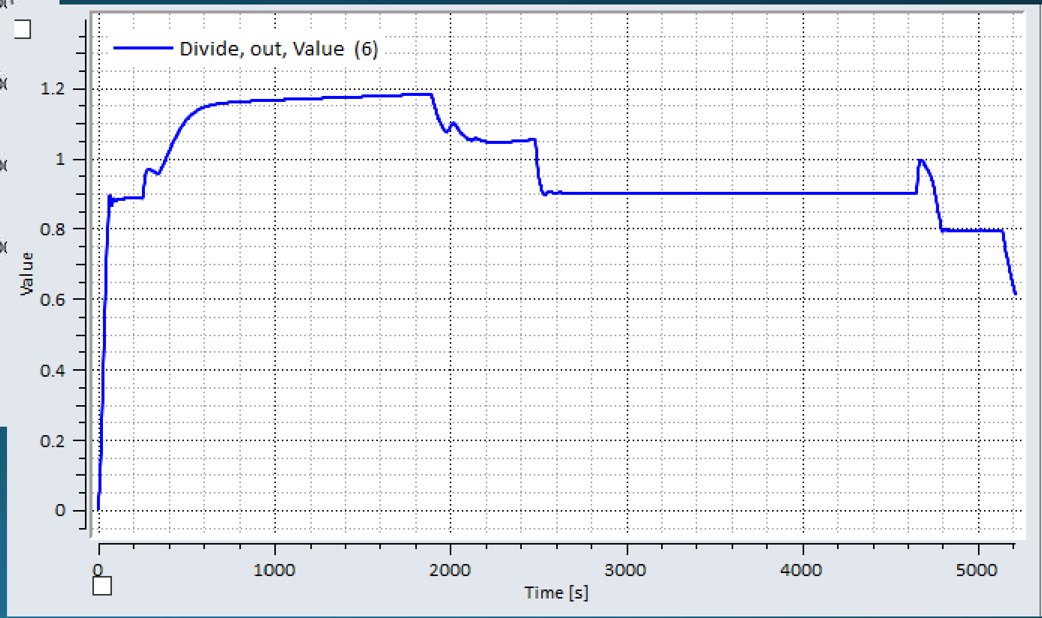



Trajectory

Mach number



# Conclusions

- In this paper an efficient generic model to model trans-sonic and supersonic aerodynamic characteristics is presented.
- Different models for different parts of the envelope has been combined into continuous functions that can be used for flight simulation already in conceptual phase.
- One conclusion is that supersonic induced drag can be substantial and needs to be considered in conceptual design. Even though the velocity is high, the induced drag coefficient goes up when leading edge suction is lost.
- In addition the neutral point is moved backwards increasing the need for trim that is further increasing the induced drag.