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Failure induced by instability in structural composites under longitudinal compression

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OVERVIEW

- Objectives
- Representative volume and matrix shear strain
- Equilibrium equation in the loaded configuration
- The matrix cohesive model
- Incremental model and the finite element context
- A simple numerical result
- Conclusions

OBJECTIVES

Originally microbuckling was explicitly modeled since it is recognized to be the main damage mechanism in longitudinal failure of composites. However, more recently alternative models have sprung up not considering microbuckling, but rather using only damage parameters instead. The motivation of this work is to merge both approaches

Develop a physically based damage propagation model applicable to the study of failure mechanism in composites induced by longitudinal compression

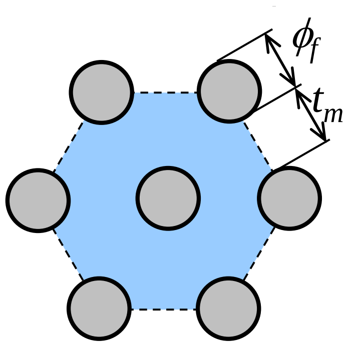
The kinematics of the equilibrium of a representative volume of a ply material within the damaged region of the laminate is the starting point

The damage propagation model is based on an energy principle, quantified by the critical energy release rate associated to matrix cracking

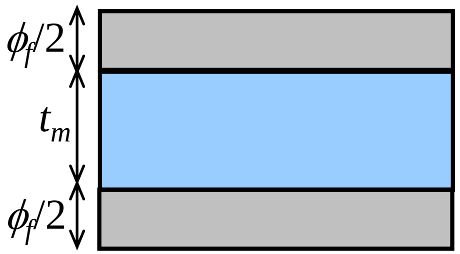
The model proposed should capture onset of the formation of kink bands and its growth

The model can be implemented within the context of finite element analyses

REPRESENTATIVE MODEL AND MATRIX SHEAR STRAIN



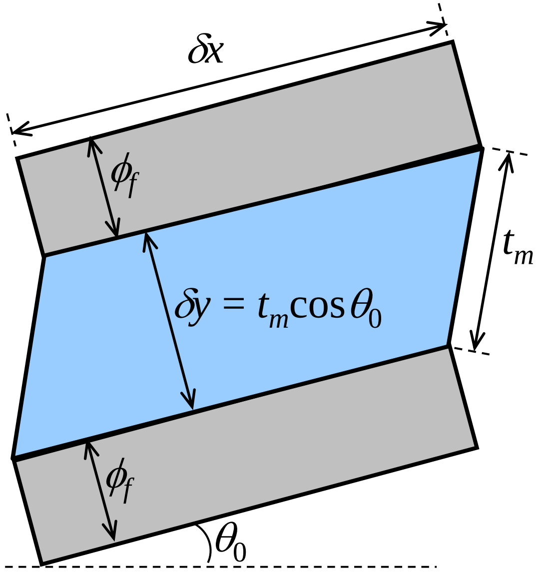
3D situation



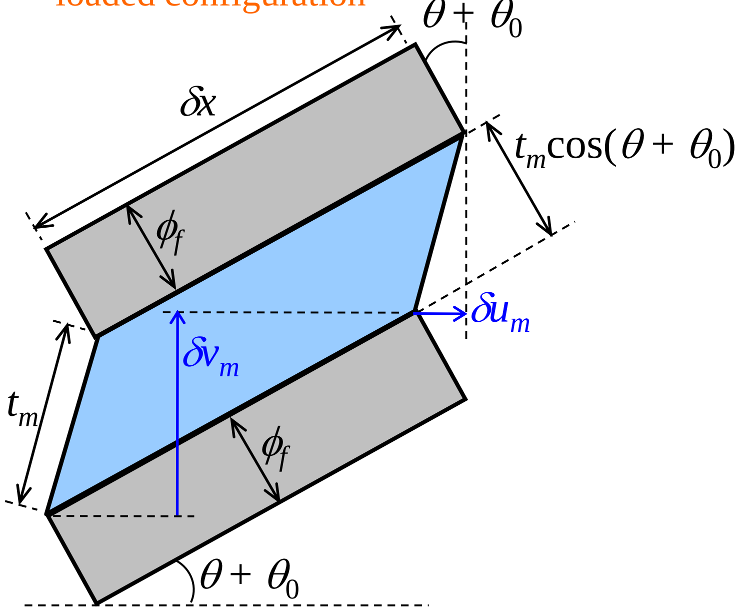
2D situation

$$v_f^{2D} = \frac{\phi_f}{\phi_f + t_m}$$

unloaded configuration



loaded configuration



$\delta y = t_m \cos\theta_0$

$\delta u_m = \phi_f[\sin(\theta + \theta_0) - \sin\theta_0]$

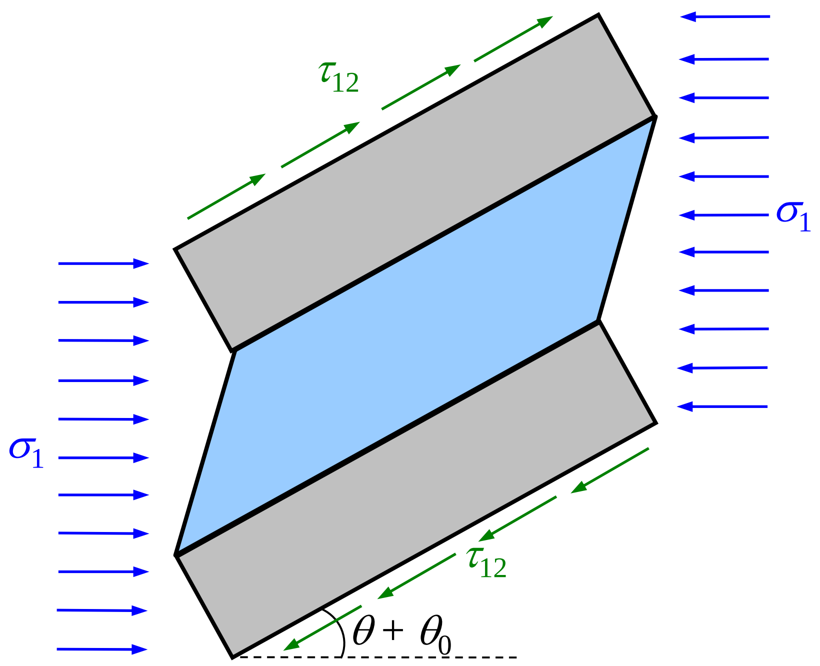
$\delta v_m = \delta x[\sin(\theta + \theta_0) - \sin\theta_0]$

$$\gamma_m = \frac{\delta u_m}{\delta y} + \frac{\delta v_m}{\delta x}$$

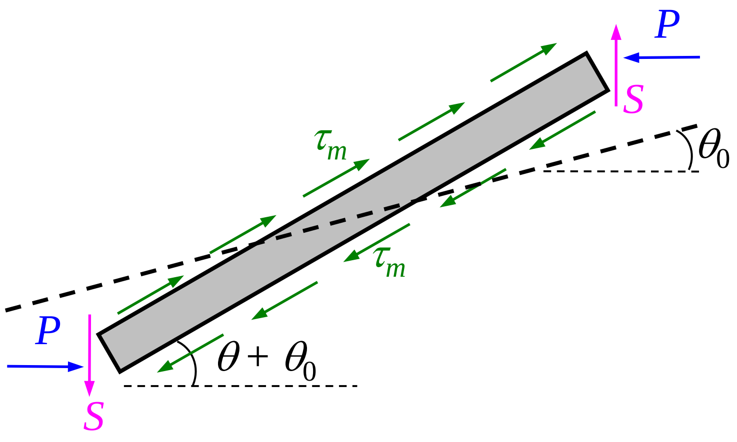
$$\gamma_m = \frac{\phi_f[\sin(\theta+\theta_0) - \sin\theta_0]}{t_m \cos\theta_0} + \frac{\delta x[\sin(\theta+\theta_0) - \sin\theta_0]}{\delta x} = \left(1 + \frac{\phi_f}{t_m \cos\theta_0}\right)[\sin(\theta+\theta_0) - \sin\theta_0] \approx \left(1 + \frac{\phi_f}{t_m}\right)\theta = \frac{\theta}{1 - v_f^{2D}}$$

$$\tau_m = G_m \gamma_m = (1 - v_f^{2D}) G_m^{2D} \frac{\theta}{1 - v_f^{2D}} = G_m^{2D} \theta$$

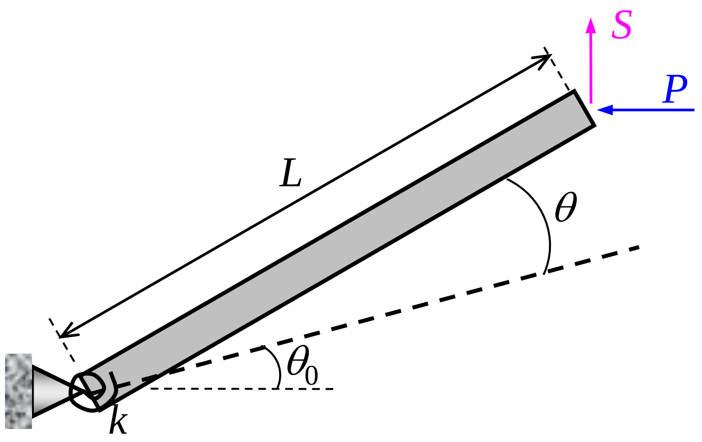
EQUILIBRIUM EQUATION IN THE LOADED CONFIGURATION



Homogenized stresses



Free body diagram of a fiber
in the loaded configuration



Initially displaced rigid bar

$$P = \frac{\sigma_1}{L v_f^{2D}} \quad , \quad S = \frac{\tau_{12}}{L} \quad , \quad k = \frac{G_m^{2D}}{L}$$

$$P = \sigma_f \phi_f = \sigma_1(\phi_f + t_m)$$

$$S = \tau_f \phi_f = \tau_{12} \phi_f$$

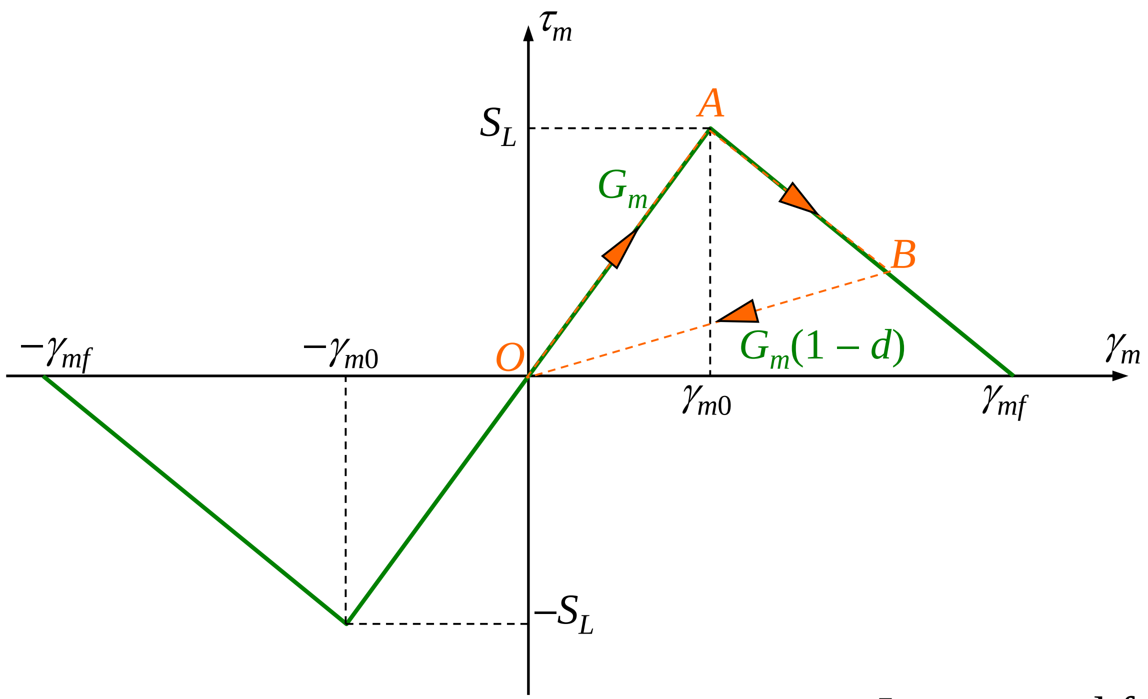
Moment equilibrium:

$$P \delta x \sin(\theta + \theta_0) + S \delta x \cos(\theta + \theta_0) = \tau_m \phi_f \delta x$$

$$\frac{\sigma_1}{v_f^{2D}} \sin(\theta + \theta_0) + \tau_{12} \cos(\theta + \theta_0) = \tau_m$$

Classical theory of micromechanics $\begin{cases} \sigma_f \phi_f = \sigma_1(\phi_f + t_m) \\ \tau_f = \tau_{12} \end{cases}$

THE MATRIX COHESIVE MODEL



$$S_L \frac{\gamma_{mf}-\gamma_m}{\gamma_{mf}-\gamma_{m0}} = G_m(1-d)\gamma_m \quad , \quad S_L = G_m\gamma_{m0}$$

$$d = \frac{\gamma_{mf}}{\gamma_{mf}-\gamma_{m0}}\left(1-\frac{\gamma_{m0}}{\gamma_m}\right)$$

Incremental form: $\tau_m = G_m(1-d)\gamma_m \quad \Rightarrow \quad \Delta\tau_m = G_m(1-d)\Delta\gamma_m - G_m\gamma_m\Delta d$

INCREMENTAL MODEL AND THE FINITE ELEMENT CONTEXT

Halpin-Tsai rules of mixture

$$E_1 = E_f v_f^{2D} + E_m(1-v_f^{2D}) \qquad \nu_{12} = \nu_f v_f^{2D} + \nu_m(1-v_f^{2D}) \qquad \xi = 1+40(v_f^{2D})^{10} \qquad \eta_E = \frac{(E_f/E_m)-1}{(E_f/E_m)+\xi}$$

$$E_2 = E_m \frac{1+\xi\eta_E v_f^{2D}}{1-\eta_E v_f^{2D}} \qquad \eta_G = \frac{(G_f/G_m)-1}{(G_f/G_m)+\xi} \qquad G_{12} = G_m \frac{1+\xi\eta_G v_f^{2D}}{1-\eta_G v_f^{2D}}$$

Ply constitutive equation

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \frac{1}{1-\nu_{12}\nu_{21}}\begin{bmatrix}E_1 & \nu_{12}E_2 & 0\\ \nu_{21}E_1 & E_2 & 0\\ 0 & 0 & (1-\nu_{12}\nu_{21})G_{12}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}$$

$$\begin{Bmatrix}\Delta\sigma_1\\ \Delta\sigma_2\\ \Delta\tau_{12}\end{Bmatrix} = \frac{1}{1-\nu_{12}\nu_{21}}\begin{bmatrix}E_1 & \nu_{12}E_2 & 0\\ \nu_{21}E_1 & E_2 & 0\\ 0 & 0 & (1-\nu_{12}\nu_{21})G_{12}\end{bmatrix}\begin{Bmatrix}\Delta\varepsilon_1\\ \Delta\varepsilon_2\\ \Delta\gamma_{12}\end{Bmatrix} + \begin{Bmatrix}0\\ 0\\ \gamma_{12}\Delta G_{12}\end{Bmatrix}$$

INCREMENTAL MODEL AND THE FINITE ELEMENT CONTEXT

Weak form equilibrium equation at time step *n*

$$\sum_{e=1}^{N_e}\int_{V_e}\delta\mathbf{q}_e^T\mathbf{B}_e^T\boldsymbol{\sigma}_e^n dV_e=\sum_{e=1}^{N_e}\int_{V_e}\delta\mathbf{q}_e^T\mathbf{f}_e^n dV_e$$

Weak form equilibrium equation at time step $n+1$

$$\sum_{e=1}^{N_e}\int_{V_e}\delta\mathbf{q}_e^T\mathbf{B}_e^T\boldsymbol{\sigma}_e^{n+1} dV_e=\sum_{e=1}^{N_e}\int_{V_e}\delta\mathbf{q}_e^T\mathbf{f}_e^{n+1} dV_e$$

$$\xrightarrow{\text{Subtraction}}$$

$$\boldsymbol{\sigma}_e^{n+1}=\boldsymbol{\sigma}_e^{n}+\Delta\boldsymbol{\sigma}_e^{n+1}$$

$$\sum_{e=1}^{N_e}\int_{V_e}\mathbf{B}_e^T\Delta\boldsymbol{\sigma}_e^{n+1} dV_e=\mathbf{f}_{ext}^{n+1}-\mathbf{f}_{ext}^{n}=\Delta\mathbf{f}_{ext}$$

Incremental forms

$$\begin{cases}\gamma_m=\left(1+\dfrac{\phi_f}{t_m\cos\theta_0}\right)[\sin(\theta+\theta_0)-\sin\theta_0] \;\Rightarrow\; \Delta\gamma_m=\left(1+\dfrac{\phi_f}{t_m\cos\theta_0}\right)\Delta\theta\cos(\theta+\theta_0)\\[2ex] \dfrac{\sigma_1}{v_f^{2D}}\sin(\theta+\theta_0)+\tau_{12}\cos(\theta+\theta_0)=\tau_m \;\Rightarrow\; \dfrac{\Delta}{v_f^{2D}}\sin(\theta+\theta_0)+\Delta\tau_{12}\cos(\theta+\theta_0)+\Delta\theta\dfrac{\sigma_1}{v_f^{2D}}\cos(\theta+\theta_0)-\Delta\theta\tau_{12}\sin(\theta+\theta_0)=\Delta\tau_m\end{cases}$$

INCREMENTAL MODEL AND THE FINITE ELEMENT CONTEXT

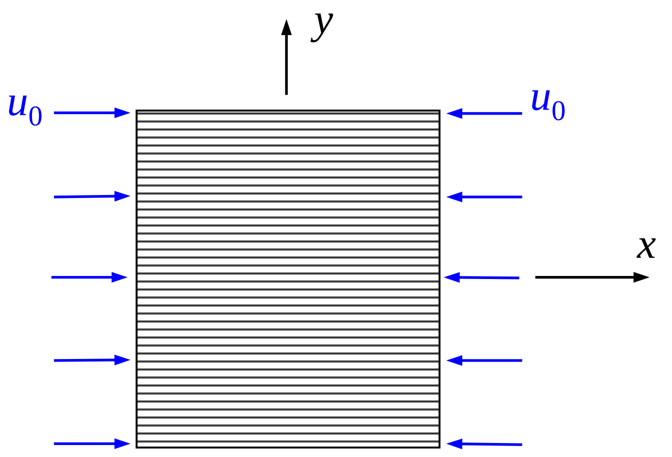
In the incremental form

$$\frac{\Delta\sigma_1}{v_f^{2D}}\sin(\theta+\theta_0)+\Delta\tau_{12}\cos(\theta+\theta_0)+\Delta\theta\frac{\sigma_1}{v_f^{2D}}\cos(\theta+\theta_0)-\Delta\theta\tau_{12}\sin(\theta+\theta_0)=$$

$$G_m(1-d)\left(1+\frac{\phi_f}{t_m\cos\theta_0}\right)\Delta\theta\cos(\theta+\theta_0)-G_m\left(1+\frac{\phi_f}{t_m\cos\theta_0}\right)[\sin(\theta+\theta_0)-\sin\theta_0]\Delta d$$

Given the stress and damage increments ($\Delta\sigma_1$, $\Delta\tau_{12}$, Δd), $\Delta\theta$ can be computed and the fiber misalignment is updated to $\theta+\Delta\theta$

A SIMPLE NUMERICAL RESULT



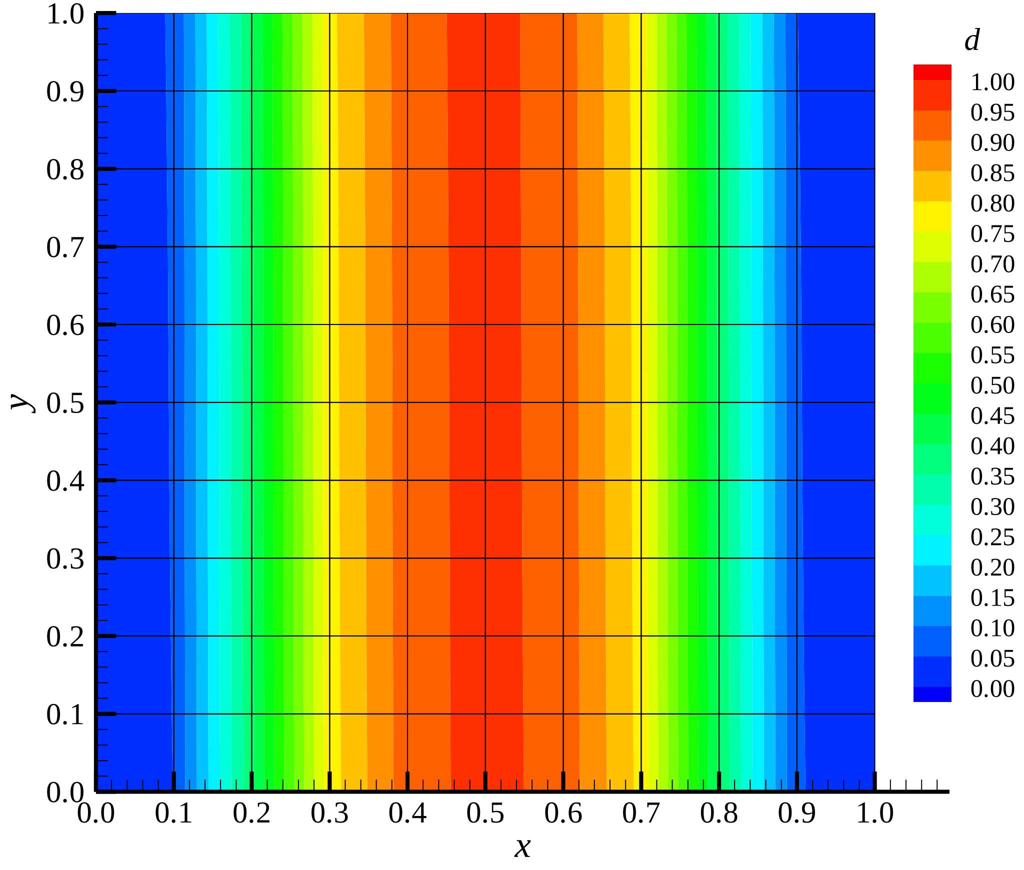
Single layer [0°] specimen:
1 m × 1 m × 0.125 mm

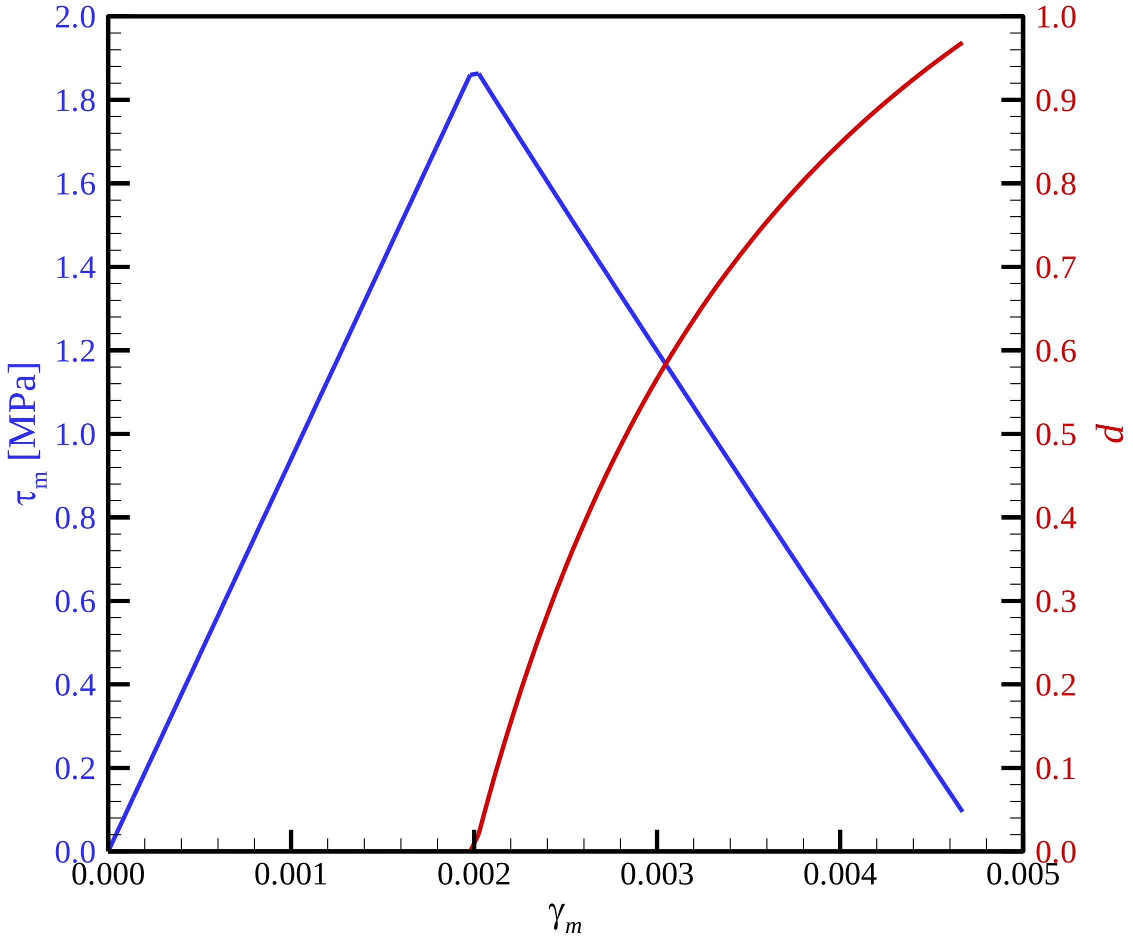
Fiber volume fraction:
60%

Initial fiber misalignment:
$\theta_o = 3.5^o \cos(\pi x/L)$

property	matrix	fiber
E_{11} [GPa]	2.6	240
E_{22} [GPa]	2.6	240
ν_{12}	0.38	0.22
G_{12} [GPa]	0.94	98.36
γ_{m0}	0.002	–
γ_{mf}	0.005	–

A SIMPLE NUMERICAL RESULT





CONCLUSIONS

The matrix is degraded as damage progresses induced by loss of stability

Only two parameters are required: γ_{m0} and γ_{mf}

Commercial finite element packages currently do not possess damage modeling capabilities well suited for prediction of failure under longitudinal compression since microbuckling and kink-band formation are neglected. Coupling present model and commercial FE code is a possibility, but element deletion strategy must be devised.

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