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Flexible Aircraft Modeling Using Multibody Dynamics and Unsteady Aerodynamics

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Overview

Motivation

- Aircraft flexibility effects are still under research Kalthof, 2014
- Increasingly aircraft are designed to be more flexible Vepa, 2014
- Exploring some flexibility benefits so as load alleviation and passengers' comfort Kalthof, 2014
- Potential application of aeroelastic analyzes in preliminary design phases Hofstee et al., 2003

Goals

Wider research challenge: Model flexible aircraft with morphing wings;

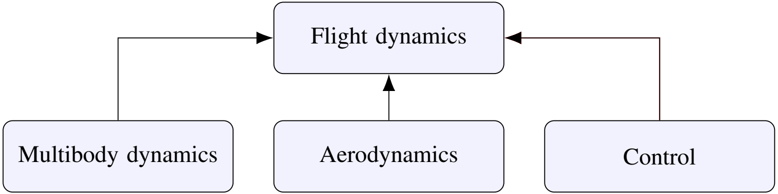
- At the moment, the work is divided into two main tasks:
  - ◇ control of trailing edge morphing
  - ◇ simulation of flexible aircraft

with a multibody approach

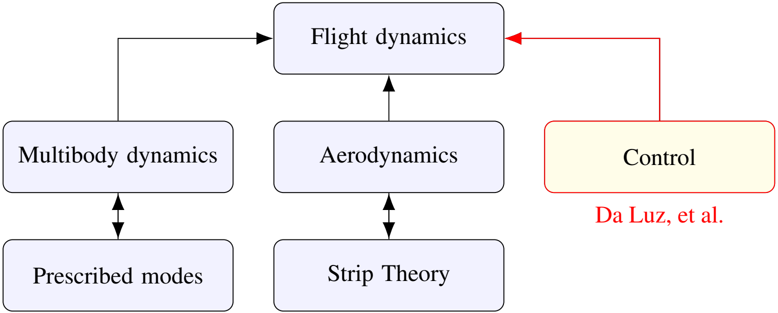
Present work goal

Establish a framework for simulation of morphing wings considering flight dynamics of flexible aircraft

Proposed framework



Proposed framework



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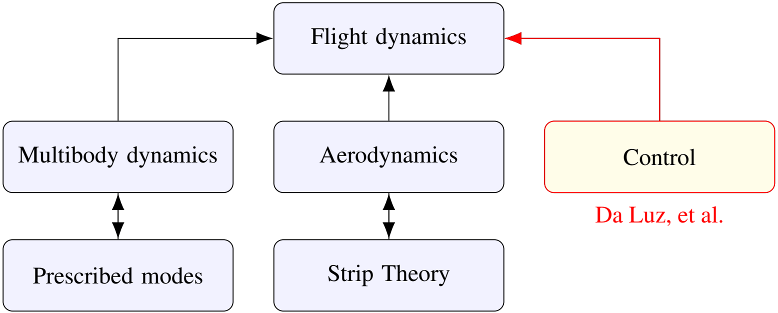


Airframe dynamics

Multibody dynamics with flexible bodies, based on formulation given by Shabana, 2013;

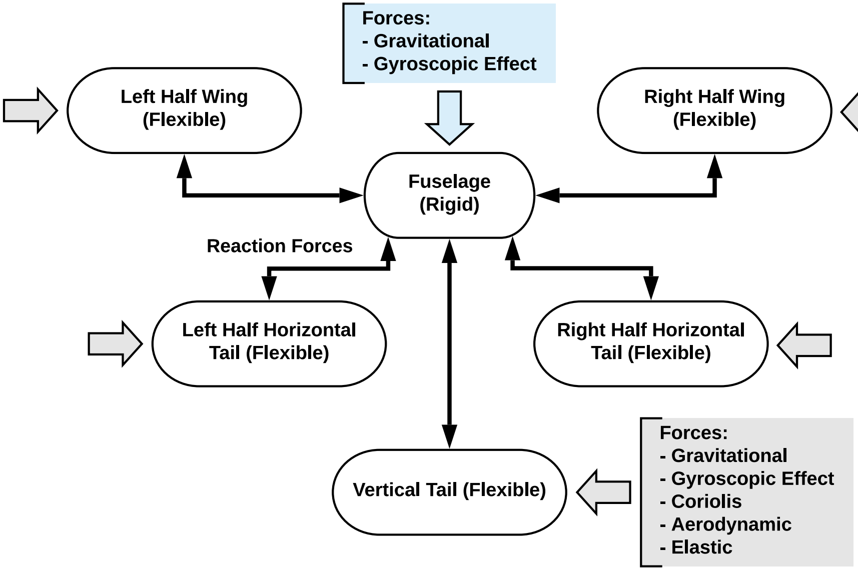
Structural model

Rayleigh-Ritz, prescribed modes, such as given in Rao, 2018, and adapted from Shabana, 2013;

Aerodynamic model

Modified Strip Theory, **barmby** approach, adapted from implementation by Pogorzelski, 2010

Multibody modeling



Dinâmica de Multicorpos

Generalized mass matrix:

$$[\mathbf{m}] = \begin{bmatrix} [\mathbf{m}]^1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & [\mathbf{m}]^2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & [\mathbf{m}]^{N_b} \end{bmatrix}$$

Generalized accelerations:

$$\{\ddot{\mathbf{q}}\} = \begin{Bmatrix} \{\ddot{\mathbf{q}}\}^1 \\ \{\ddot{\mathbf{q}}\}^2 \\ \vdots \\ \{\ddot{\mathbf{q}}\}^{N_b} \end{Bmatrix}$$

Body constraints:

$$\mathbf{C}(\mathbf{q}, t) = \begin{Bmatrix} \{\mathbf{R}\}^1 - \{\mathbf{R}\}^2 \\ \{\mathbf{\Theta}\}^1 - \{\mathbf{\Theta}\}^2 \\ \vdots \\ \{\mathbf{R}\}^1 - \{\mathbf{R}\}^{N_b} \\ \{\mathbf{\Theta}\}^1 - \{\mathbf{\Theta}\}^{N_b} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}$$

Stiffness given by prescribed modes.

Modal shapes for cantilever beam

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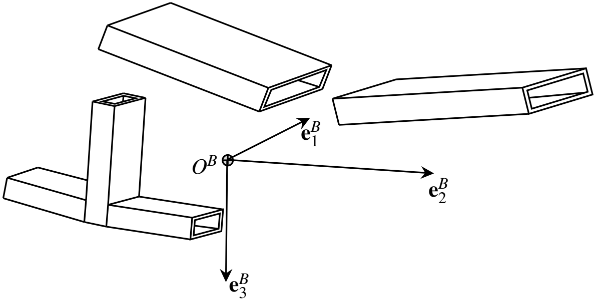
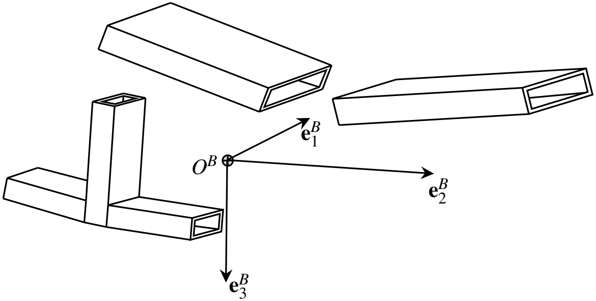
- Euler-Bernoulli
- Saint-Venant
- Rayleigh-Ritz method

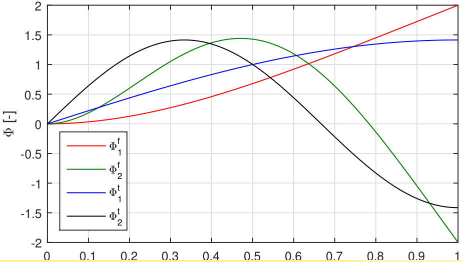


Modal shapes for cantilever beam

- Euler-Bernoulli
- Saint-Venant
- Rayleigh-Ritz method

$$\int_0^1 \Phi_i(\xi)\Phi_j(\xi)\, d\xi = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$$





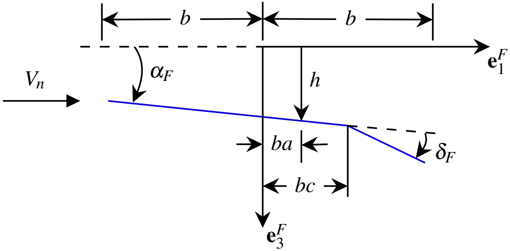
$$\bar{\bar{\mathbf{u}}}_f = \begin{bmatrix} u_1^f \\ u_2^f \\ u_3^f \end{bmatrix} = \begin{bmatrix} 0 & 0 & x_3\Phi_1^t & x_3\Phi_2^t \\ -x_3\dfrac{d\Phi_1^f}{dx_2} & -x_3\dfrac{d\Phi_2^f}{dx_2} & 0 & 0 \\ \Phi_1^f & \Phi_2^f & -x_1\Phi_1^t & -x_1\Phi_2^t \end{bmatrix} \begin{bmatrix} q_1^f \\ q_2^f \\ q_1^t \\ q_2^t \end{bmatrix}$$

Aerodynamic model - strip theory

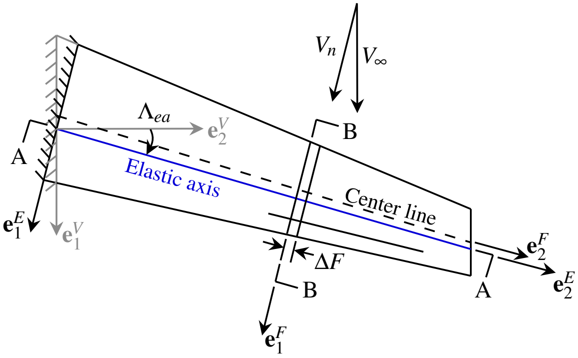
Modified Strip Theory (MST)

- Unsteady strip theory
  - ◇ good computational cost / precision rate

- Unsteady modified strip theory (UMST) - **barmby**



Modification for swept wings



Aerodynamic model - strip theory

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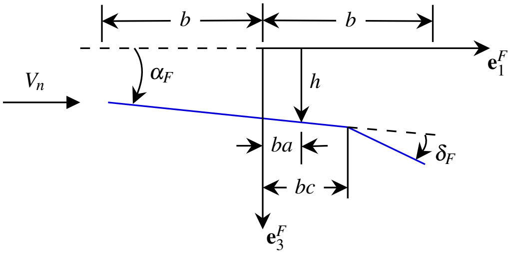
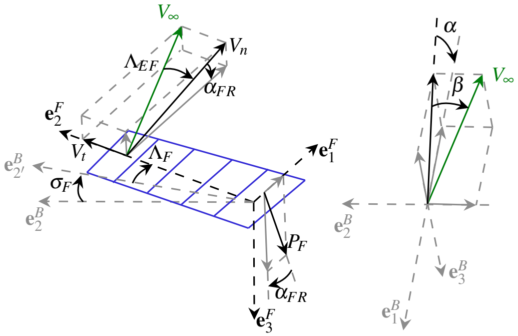


Rigid body motion corrections



Forces on each strip are transformed into forces and moments in frequency domain on the Body Reference Systems

$$\Rightarrow \overline{\mathbf{F}}_a^F(ik) \qquad \Rightarrow \overline{\boldsymbol{\tau}}_a^F(ik)$$

Aerodynamic Influence Coefficients Matrix **AIC**

$$\overline{\mathbf{Q}}_a^F(ik) \triangleq \begin{bmatrix} \overline{\mathbf{F}}_a^F \\ \overline{\boldsymbol{\tau}}_a^F \end{bmatrix} = \mathbf{AIC}^F(ik)\Delta\mathbf{x}_a$$

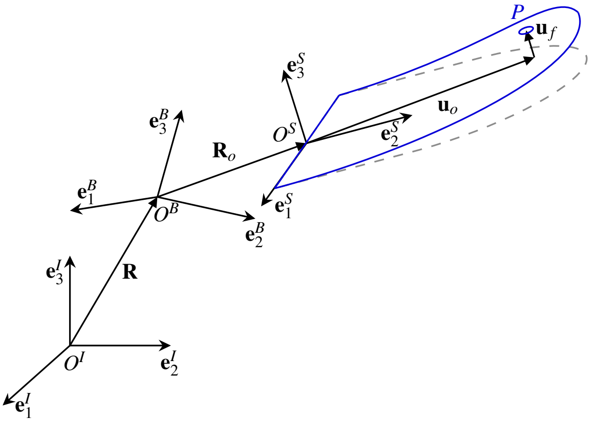
Total **AIC** matrix for multibodies:

$$\mathbf{AIC}_{tot}(ik) = \begin{bmatrix} \sum_{F=1}^{nF1} [\mathbf{AIC}^F]_1 \\ \sum_{F=1}^{nF2} [\mathbf{AIC}^F]_2 \\ \vdots \\ \sum_{F=1}^{nF5} [\mathbf{AIC}^F]_5 \end{bmatrix}$$

Rational function approximation - Roger method.
Use Padé polynomials to obtain approximate **AIC**:

$$\mathbf{AIC}_{ap}(ik) = [\mathbf{A}_0] + ik[\mathbf{A}_1] + (ik)^2[\mathbf{A}_2] + \sum_{n=1}^{n_{lag}} \frac{ik[\mathbf{A}_{n+2}]}{ik+\beta_n}$$

Displacement kinematics

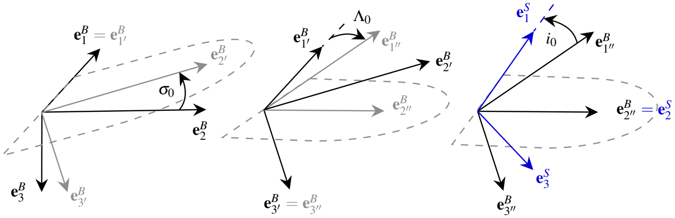


$$\mathbf{r} = \mathbf{r} + \mathbf{A}_{BI}\bar{\mathbf{u}}$$

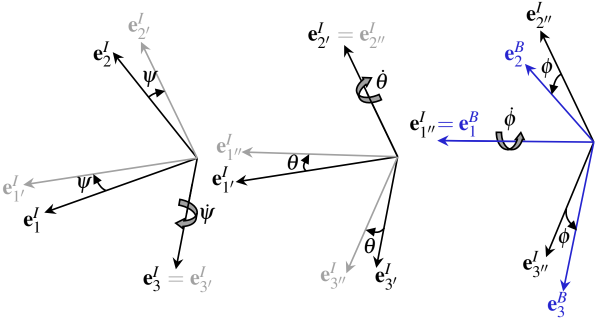
$$\bar{\mathbf{u}} = \bar{\mathbf{r}}_o + \mathbf{A}_{SB}(\bar{\bar{\mathbf{u}}}_o + \bar{\bar{\mathbf{u}}}_f)$$

$$\bar{\bar{\mathbf{u}}}_f = \mathbf{S}\mathbf{q}_f$$

Atitude Euler and coordinate transformations



$$\mathbf{A}_{BS} = \mathbf{C}_2(i_0)\mathbf{C}_3(\Lambda_0)\mathbf{C}_1(-\sigma_0)$$



$$\mathbf{A}_{IB} = \mathbf{C}_1(\phi)\mathbf{C}_2(\theta)\mathbf{C}_3(\psi)$$

Atitude Euler angles and Kinematics

Translation:

$$\dot{\mathbf{r}} = \mathbf{A}_{BI}\dot{\bar{\mathbf{r}}}$$

Rotation:

$$\overline{\boldsymbol{\omega}} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \mathbf{C}_1(\phi)\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \mathbf{C}_1(\phi)\mathbf{C}_2(\theta)\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

$$\dot{\boldsymbol{\Theta}} = [\overline{\mathbf{G}}]^{-1}\ \overline{\boldsymbol{\omega}}$$

$$[\overline{\mathbf{G}}]^{-1} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sec(\theta)\sin(\phi) & \cos(\phi)\sec(\theta) \end{bmatrix}$$

Generalized Newton-Euler equations

$$\begin{bmatrix} \mathbf{m}_{RR} & \tilde{\bar{\mathbf{S}}}_t^T & \bar{\mathbf{S}} \\ \tilde{\bar{\mathbf{S}}}_t & \bar{\mathbf{I}}_{\Theta\Theta} & \bar{\mathbf{I}}_{\Theta f} \\ \bar{\mathbf{S}}^T & \bar{\mathbf{I}}_{\Theta f}^T & \mathbf{m}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\bar{\mathbf{r}}} \\ \bar{\boldsymbol{\Omega}} \\ \ddot{\mathbf{q}}_f \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{Q}}_v^R \\ \mathbf{Q}_v^\Omega \\ \mathbf{Q}_v^f \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{Q}_s^f \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{F}}_a + m\bar{\mathbf{g}} \\ \tilde{\bar{\mathbf{u}}}(\bar{\mathbf{F}}_a + m\bar{\mathbf{g}}) + \bar{\boldsymbol{\tau}}_a \\ \mathbf{S}^T\, \bar{\bar{\mathbf{F}}}_a + \dfrac{\partial \mathbf{S}^T}{\partial x_1}\boldsymbol{\Pi}_{23}\bar{\bar{\boldsymbol{\tau}}}_a \end{bmatrix}$$

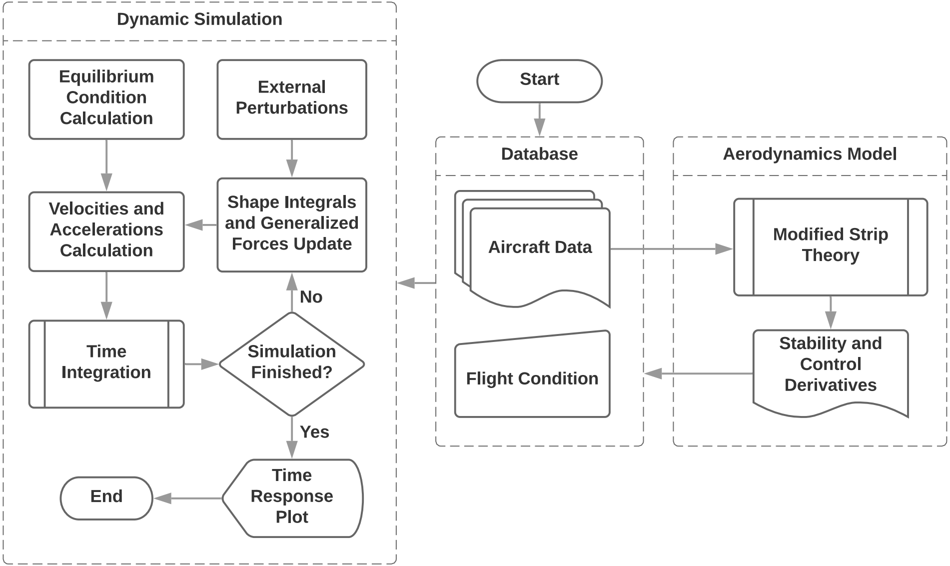
$$\mathbf{Q}_s^f = -\mathbf{K}_{ff}\mathbf{q}_f - \mathbf{D}_{ff}\dot{\mathbf{q}}_f$$

$$\begin{bmatrix} \bar{\mathbf{Q}}_v^R \\ \mathbf{Q}_v^\Omega \\ \mathbf{Q}_v^f \end{bmatrix} = \begin{bmatrix} \tilde{\bar{\boldsymbol{\omega}}}\,\tilde{\bar{\boldsymbol{\omega}}}\,\bar{\mathbf{S}}_t - 2\tilde{\bar{\boldsymbol{\omega}}}\mathbf{A}_{SB}\bar{\mathbf{S}}\dot{\mathbf{q}}_f \\ -\tilde{\bar{\boldsymbol{\omega}}}\,\bar{\mathbf{I}}_{\Theta\Theta}\bar{\boldsymbol{\omega}} - \dot{\bar{\mathbf{I}}}_{\Theta\Theta}\bar{\boldsymbol{\omega}} - \tilde{\bar{\boldsymbol{\omega}}}\,\bar{\mathbf{I}}_{\Theta f}\dot{\mathbf{q}}_f \\ -\int_V \rho \mathbf{S}^T \mathbf{A}_{SB}^T \tilde{\bar{\boldsymbol{\omega}}}(\tilde{\bar{\boldsymbol{\omega}}}\,\bar{\mathbf{u}} + 2\mathbf{A}_{SB}\mathbf{S}\dot{\mathbf{q}}_f)\ dV \end{bmatrix}$$

Final multibody equations

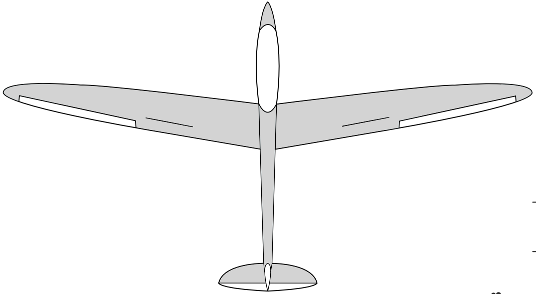
$$\begin{bmatrix} \mathbf{m} & \mathbf{C}_\mathbf{q}^T \\ \mathbf{C}_\mathbf{q} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_v + \mathbf{Q}_s + \mathbf{Q}_e \\ \mathbf{0} \end{bmatrix}$$

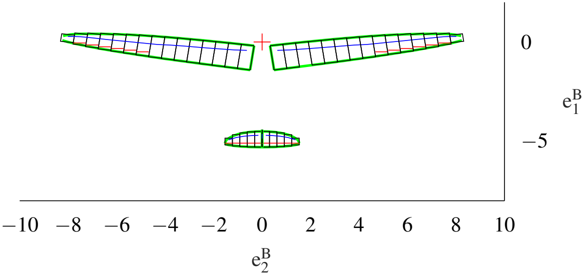
Numerical framework

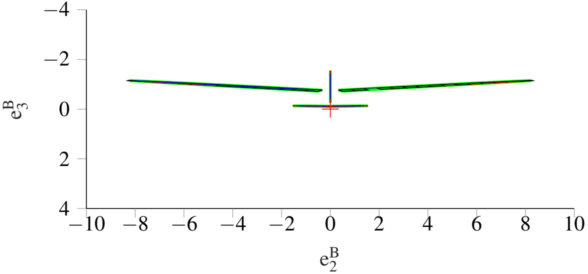


Reference Aircraft and strips model

P1- Glider - modified







Study: equilibrium condition

Parameters	Rigid body	Flexible body
Velocity, V $[km/h]$	80	80
Altitude, H $[m]$	2000	2000
Angle of Attack, α $[^\circ]$	4.1614	4.0961
Pitch angle, θ $[^\circ]$	2.6284	2.5871
Roll angle, ϕ $[^\circ]$	0	0
Elevator deflection, δ_e $[^\circ]$	-5.2204	-5.0786

Study: equilibrium condition

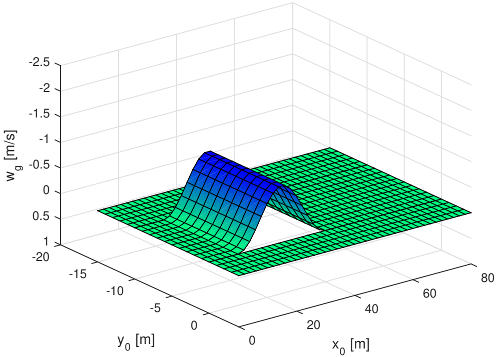
Elastic coordinate		Rigid body	Flexible body
Bending 1 RW, q_{11}^f	$[m]$	0	-0.04650
Bending 2 RW, q_{21}^f	$[m]$	0	-0.00248
Torsion 1 RW, q_{11}^t	$[^\circ]$	0	0.00010
Torsion 2 RW, q_{21}^t	$[^\circ]$	0	0.00001
Bending 1 LW, q_{12}^f	$[m]$	0	-0.04650
Bending 2 LW, q_{22}^f	$[m]$	0	-0.00248
Torsion 1 LW, q_{12}^t	$[^\circ]$	0	-0.00010
Torsion 2 LW, q_{22}^t	$[^\circ]$	0	-0.00001

Elastic coordinate		Rigid body	Flexible body
Bending 1 RHT, q_{13}^f	$[m]$	0	-0.000118
Bending 2 RHT, q_{23}^f	$[m]$	0	-0.000006
Torsion 1 RHT, q_{13}^t	$[^\circ]$	0	0.000063
Torsion 2 RHT, q_{23}^t	$[^\circ]$	0	0.000007
Bending 1 LHT, q_{14}^f	$[m]$	0	-0.000118
Bending 2 LHT, q_{24}^f	$[m]$	0	-0.000006
Torsion 1 LHT, q_{14}^t	$[^\circ]$	0	-0.000063
Torsion 2 LHT, q_{24}^t	$[^\circ]$	0	-0.000007
Bending 1 VT, q_{15}^f	$[m]$	0	0
Bending 2 VT, q_{25}^f	$[m]$	0	0
Torsion 1 VT, q_{15}^t	$[^\circ]$	0	0
Torsion 2 VT, q_{25}^t	$[^\circ]$	0	0

Study: Gust profile

Applying gust to the left wing only:

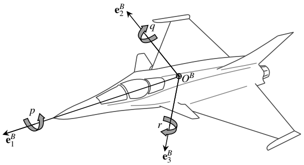
$$w_g(x_0, y_0) = \begin{cases} -\dfrac{V_{0w}}{2}\left[1 - cos\left(\dfrac{\pi(x_0 - x_{0w})}{H_w}\right)\right], & \begin{array}{c} x_{0w} \leq x_0 \leq x_{fw} \\ y_{0w} \leq y_0 \leq y_{fw} \end{array} \\ 0, & \text{otherwise} \end{cases}$$

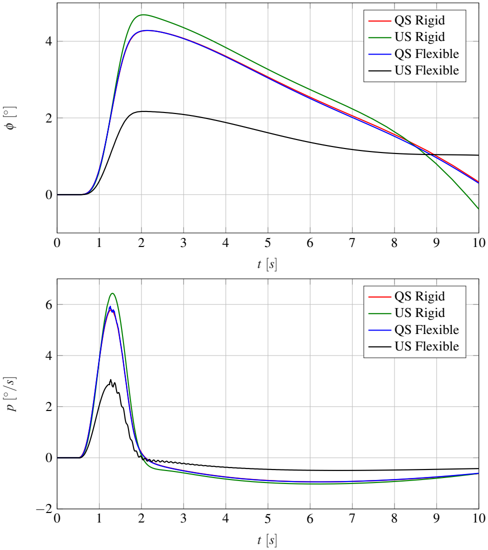


Study: Gust profile - models comparison

Comparison between:

case	aerodynamic model	aircraft
QSR	quasi-steady	rigid
QSF	quasi-steady	Flexible
USR	unsteady	rigid
USF	unsteady	Flexible





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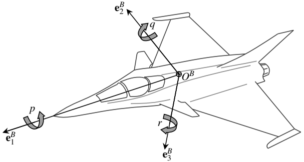

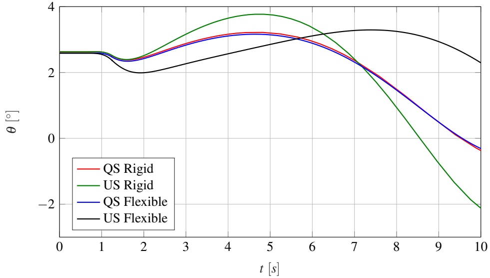

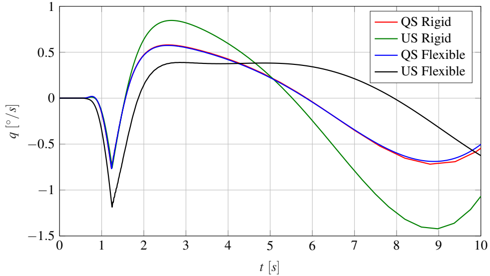

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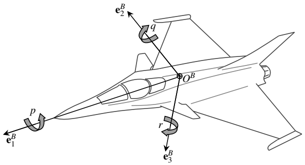
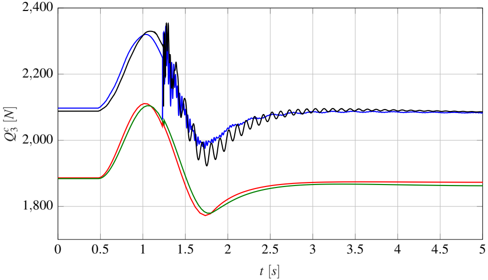
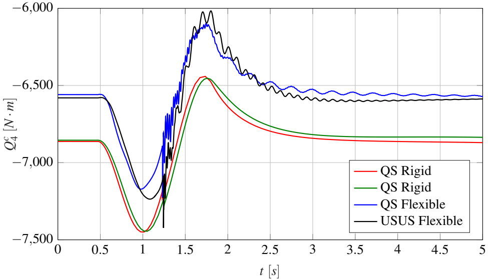
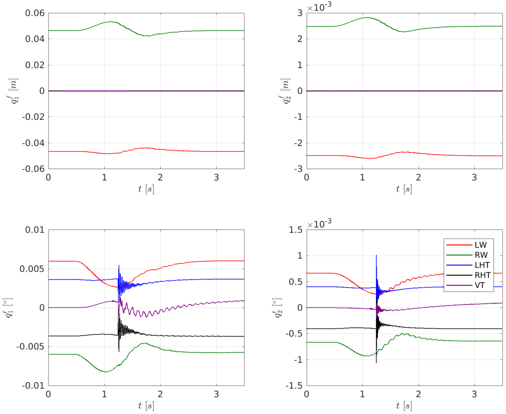




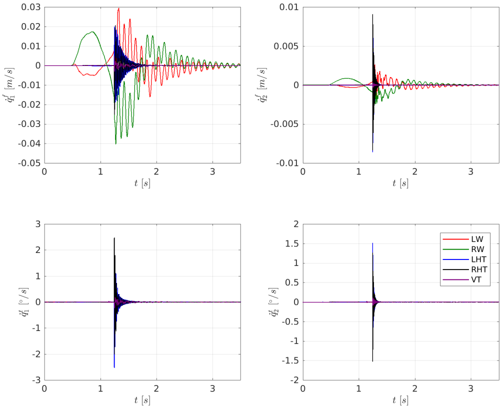


Generalized elastic coordinates

Bending



Torsion



Conclusions from current study

- Achieved: flight mechanics model for flexible aircraft using multibody dynamics
- Simplified structural model is possible when experimental data is available for calibrating beam models
- Flexibility combined with unsteady aerodynamics leads to more oscillating responses
- Framework developed allows quick simulations (mostly analytical models)
- Creates a wide range of possibilities to be explored combining this modeling with morphing wing control

Next: couple the control model implemented by da Luz, et al.

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Bibliography I

- Hofstee, J., Kier, T., Cerulli, C., and Looye, G. (2003). “A Variable, Fully Flexible Dynamic Response Tool For Special Investigations (VARLOADS)”. In: *Proceedings International Forum on Aeroelasticity and Structural Dynamics*. Amsterdan: Nederlandse Vereniging voor Luchtvaarttechniek.
- Kalthof, R. L. C. (2014). “Multibody Dynamics Modeling of Flexible Aircraft Flight Dynamics: Load Prediction in the Preliminary Design Phase”. Mestrado em Engenharia Aeroespacial. Delft: Universidade Tecnológica de Delft, p. 150.
- Pogorzelski, G. (2010). “Dinâmica de Aeronaves Flexíveis Empregando Teoria das Faixas Não-Estacionária”. Mestrado em Engenharia Aeronáutica e Mecânica. São José dos Campos: Instituto Tecnológico de Aeronáutica, p. 326.
- Rao, S. (2018). *Mechanical vibrations in SI units*. 6ed. Pearson. ISBN: 978-0-134-36130-7,1-292-17860-4,978-1-292-17860-8,9781292178615,1292178612.
- Shabana, A. (2013). *Dynamics of Multibody Systems*. 4th ed. Cambridge University Press. ISBN: 978-1-107-04265-0.

Bibliography II

Vepa, R. (2014). *Flight Dynamics, Simulation, and Control: For Rigid and Flexible Aircraft.* CRC Press. ISBN: 978-1-4665-7336-9.